







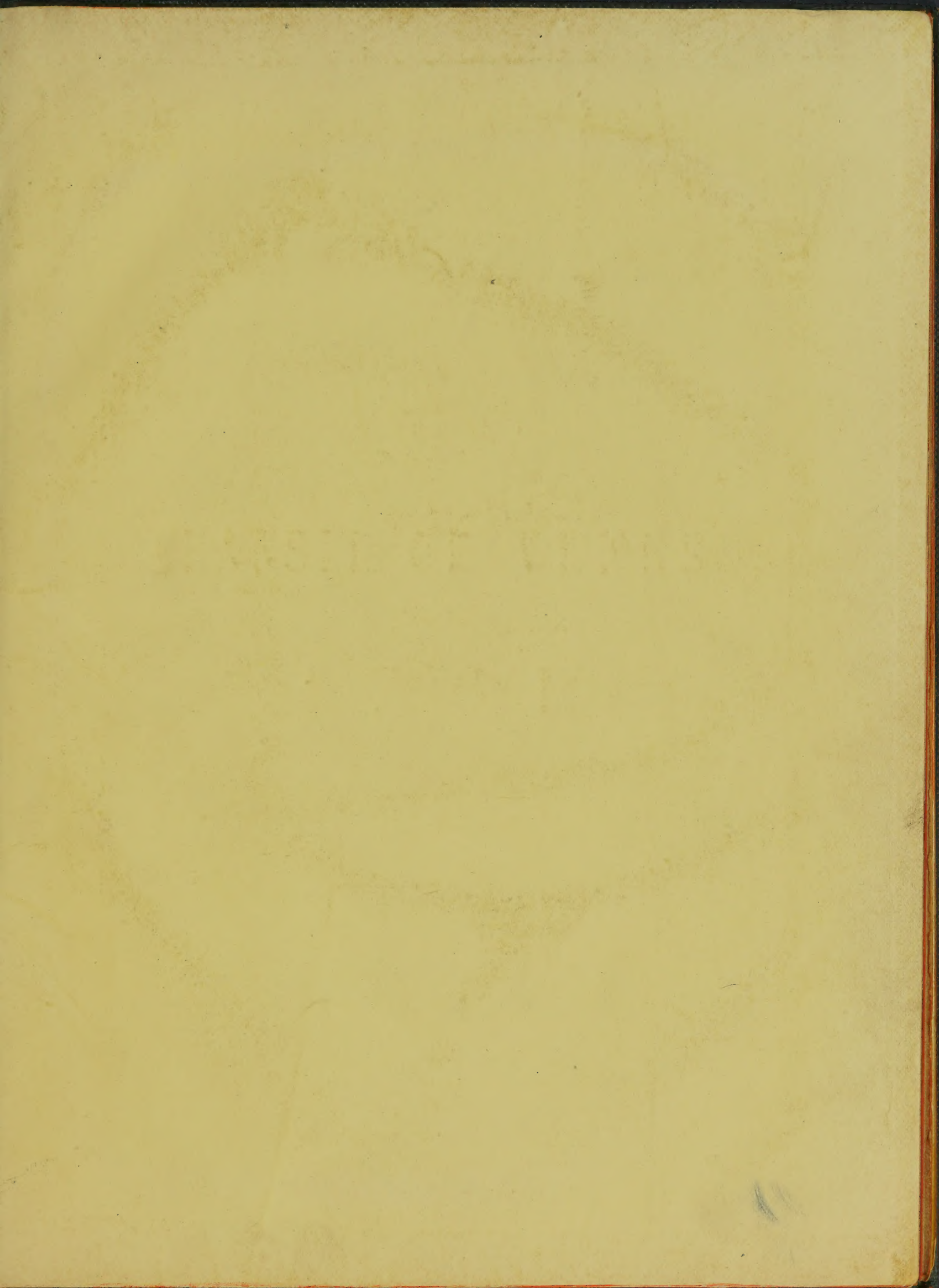


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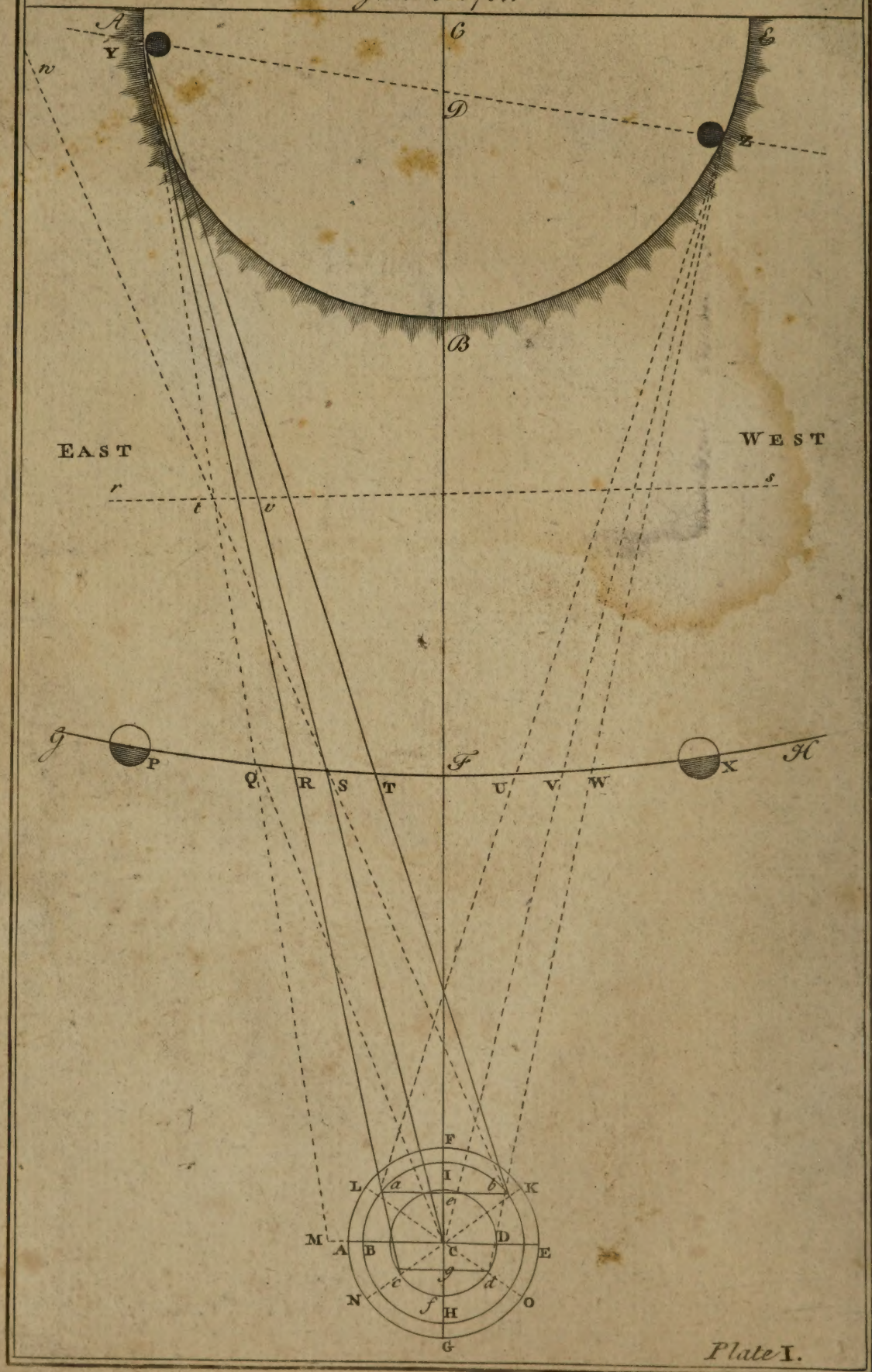
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The TRANSIT of VENUS over the SUN'S DISK
June 6. 1761.



Mr. Lintoul

VENUS in the SUN:

BEING AN

EXPLICATION of the RATIONALE

OF THAT

GREAT PHÆNOMENON;

OF THE

Several METHODS used by ASTRONOMERS for Computing
the QUANTITY and PHASES thereof;

And of the Manner of applying a

Tranfit of VENUS over the SOLAR DISK,

For the DISCOVERY of the

P A R A L L A X of the S U N ;

Settling the THEORY of that PLANET'S MOTION, and Ascertaining

T H E

DIMENSIONS of the SOLAR SYSTEM.

By BENJAMIN MARTIN.

L O N D O N :

Printed for W. OWEN, at Homer's Head, near Temple-Bar.

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T H E
P R E F A C E.

I*T is now twelve Months since I first published a Commentary on Dr. HALLEY's Dissertation on the TRANSIT OF VENUS, in the General Magazine of Arts and Sciences; but being often importuned to publish that Treatise in one entire View, I at length concluded to do it, but not without considerable Alterations and Additions thereto. For though the Doctor's Performance was a Master-Piece in its Kind, yet there is another, and, perhaps, the most genuine Method of Computing the Quantity, Phases, and various Affections of the TRANSIT, which has not, that I know of, been ever explained in English; and therefore I have here attempted it, for the sake of such as may be willing to know the most direct and Astronomical Method of Computation; I mean that of PARALLAXES; the Theory of which I have laid down in the plainest Manner I could, and have exemplified the same by adapting the Calculus to the End of the Transit, as it will be observed here in England.*

I am well satisfied, neither the Doctor's Method by Projection, nor this other by Parallaxes, can be well understood, without a proper Idea of the Theory of this Planet; which therefore I have supplied, and by that means the Rationale of Calculation will be more intelligible.

After this you will find the Use of Transit for ascertaining the Elements of the Theory of Venus's Motions, according to the Method of Parallaxes used by Mr. HORROX and Mr. CASSINI, exemplified in the Transit of 1639, now first done into English.

I have also added an Account of the Motion of the Nodes of Venus, as I find it in all the Tables hitherto published that I have seen, in order

der that the Reader might have a proper Notion of so fundamental a Principle.

As to the Geographical Principles of the Transit, I have illustrated them by the Use of the Terrestrial GLOBE, as a necessary Introduction to the whole. Nor is the Use of the Celestial GLOBE inconsiderable, as thereby any Person, curious in these Matters, may easily attain to a practical Method of determining the Angle $E S Z$ (see Figure 3 and 4. Plate III.), made by the Circle of Latitude $E R$, and the Vertical $C S N$, by which means the Position of the ECLIPTIC $T L$ will be obtained, and thereby a Representation of the solar Disk formed as large as you please for any given Place and Moment of the Transit; by this means also the several Phænomena of the Transit may be estimated by the Scale and Compasses; and a Heliometer adjusted for measuring the Distance of the Planet from the Ecliptic, at any Moment of her Passage.

INTRODUCTION,

SHEWING

The USE of the TERRESTRIAL and CELESTIAL GLOBE
in exhibiting the various Phænomena of the TRANSIT
of VENUS.

AS I conceived the most familiar and general Way of exhibiting the various Phænomena of a Planet transiting the Sun's Disk, is by means of the artificial Globes, I shall here briefly describe that Method, by way of Introduction to Dr. HALLEY's Dissertation, and what afterwards follows.

In order to this, we first consider the Place of the Sun in the Ecliptic, and his Declination for the middle Time of the Transit, and then, by rectifying the Globe, we can easily shew all those Places of the Earth where the Beginning, the Middle, and End of the Transit can be observed, and at what particular Times of the Day; as also how, by the different Directions and Motions upon contrary Parts of the Globe, the Times of the Transit will be affected, so as to be in some Places longer, and in others shorter; and lastly, what Position the Ecliptic will have in regard to a vertical Circle for any given Place, and of course what the Position of the Path of the Planet will be, in its Passage over the Sun for that particular Place; all which together will constitute a general Idea of this great and most interesting Phænomenon.

The Place of the Sun in the Ecliptic, when the Planet *Venus* will be seen to enter his Disk, is in $15^{\circ} 30'$ of *Gemini*, and of course his Declination will be near 22° ; and therefore if the Globe be rectified for that Parallel of 22° Latitude, then will it be in a Condition to exhibit all the Appearances of the Transit. This we presume the Reader knows must be done by elevating the North Pole 22° above the North Part of the Horizon.

The

The Time (reckoned at LONDON) of the Beginning of the Transit we find, by Dr. HALLEY's Tables, is about Five Minutes after II. in the Morning, on the Ninth Day of *June* next; the true Conjunction, or rather Middle of the Transit, will be about half an Hour after V.; and the End about three Quarters after VIII. So that the whole Duration will be little more than six Hours and a half.

Now to shew all the Places on the Globe to which the BEGINNING will be visible; you bring the Place (LONDON, for Instance); to the Meridian (having rectified the Globe as above directed) and then place the Hour-Index at the Hour of II. in the Morning; then turn the Globe about till the Index points to XII. at Noon, and the upper Hemisphere will then be that only, in which the Beginning of the Transit can be seen; where you will observe that all those Places which lie under the Meridian, will see *Venus* enter the *Sun's* Disk upon the Meridian at XII o'Clock. But these will be very few; viz. The Eastern Parts of *Muscovy*, and *New Britain*, near the Equator; the *Great Pacific Ocean* almost wholly occupying the Meridional Parts of the Globe. And here it is to be observed, that the *Sun* is vertical at that Time to the most Northern of those Islands called the *Ladrones*.

In the next Place, you observe all the Places of the Globe lying under the Western Semicircle of the Horizon; for to them the *Sun* will be then rising; the Reason of which is evident, because the *Sun* being over the middle Point of the Hemisphere above the Horizon, will illuminate the same throughout; and the wooden Horizon of the Globe is now, properly speaking, the Solar Horizon; and the general Meridian, in which the *Sun* is, becomes the Horizon to all those who lie under the Western Horizon of the Globe. These Countries, by the Motion of the Globe, are just entering into the illumined Hemisphere, and see *Venus* entering the *Sun's* Disk, just risen above their Eastern Horizon. These Countries, you will observe, are *Norway* and *Sweden*, *Poland*, the *Black Sea*, and all the middle Part of *Turky* in *Asia*; after which, to the Southern Parts of the Meridian, you see nothing but the *Great Eastern Ocean*.

From hence it follows, that all in that Quarter of the Globe which lieth between the Western Horizon and the Meridian, will see the Beginning of the Transit some time in the Morning, or Forenoon, and later in Proportion as they are nearer to the Meridian, at the rate of an Hour to every 15° in the Equator. So that from
hence

hence it will be very easy to know, by Inspection, the Hour of the Morning when this Phænomenon can be observed; because, on *Senex's* twelve-inch Globes, the Meridians are real Hour-Circles, being drawn through every 15° of the Equator, and consequently are 24 upon the Whole. You then cast your Eye on the Eastern Semicircle of the Horizon, and under that you see but a small Part of the inhabited Globe, *viz. Hudson's Bay*, the Western and South-West unknown Parts of *North America*, and Southern Parts of *California*. If they have ever heard of the Transit, and have any Means of viewing the *Sun*, they will see the Planet enter his Disk just as the *Sun* is setting, because at that Instant they are, by the Motion of the Globe, carried Eastward out of the enlightened Hemisphere.

It is very remarkable, that all this Quarter of the Globe, between the Meridian and Eastern Horizon, is covered with Water, except some of the unknown Parts of *America*, extending towards *Muscovy*; and therefore it is probable the Beginning of the Transit will not be observed in the Afternoon any-where, unless by those who navigate the *Great Pacific Ocean*.

It is further to be observed, that all the Parts of the World which lie on the nether Hemisphere, cannot see the Beginning of the Transit; among which, below the Western Horizon, near the North, you will observe our Island of *Great Britain*, great Part of *Europe*, all *Africa*, and almost all *America*. Thus much for the BEGINNING of the Transit.

In the next Place, to exhibit the Phænomena for the MIDDLE of the Transit. The Globe remaining rectified as before, you bring the City of *London* to the Meridian, and there holding it firmly, you set the Hour-Index to half an Hour past V. in the Morning, and then revolve the Globe till the Index points to the Hour of XII. at Noon; and thus you will have in View that Hemisphere of the Globe in which the MIDDLE of the Transit can be seen, *viz. all above the Horizon*. And to those who inhabit the Hemisphere below, which are, nearly, all the Parts of *America*, and the Western Parts of *Africa*, the Middle of the Transit will not appear.

From what we have said before, it is evident, that to all the Parts of the Globe which now lie under the general Meridian of the illumined Hemisphere, *Venus* will appear in Conjunction with the *Sun* at XII o'Clock. These are all the Midland Parts of *Asia*, from
the

the Northern Parts of *Muscovy* to the *East Indies*, near which you will see *Bencoolen*, in the Isle of *Sumatra*.

In the Western Hemisphere you will observe all the Countries in the Western Semicircle of the Horizon to which the *Sun* will rise, with *Venus* advanced to the middle Part of her Path over his Disk. These are only the Western and Southern Parts of *Africa*. In all this Quarter between the Western Horizon and the Meridian, the Middle of the Transit will be seen in the Forenoon, earlier or later, as the Places are more remote, or nearer to the Meridian. Among these you will observe our Island at a small Altitude above the Horizon, having been *risen* (as one may say) into the illumined Hemisphere but one Hour and half.

By turning the Eye to the Eastern Semicircle of the Horizon, you will find nothing but the *Great Pacific Ocean* lying under it, excepting a small Portion of the undiscovered Parts of *North America*, and some Parts of the Coast of *New Zealand*, in Southern Latitude. It is therefore probable that the Middle of the Transit will be no-where observed at Sun-set; but in all the Countries in this Quarter of the Globe it will be seen at some time or other in the Afternoon, and to most of them before IV o'Clock.

In the last Place, for the END of the Transit, which is at Three-quarters after VIII. in the Morning, we must rectify the Globe for that Time; that is, bringing *London* to the Meridian, you must set the Hour-Hand to that Time, and then revolve the Globe till it points to the upper XII. and you will have presented to your View that Hemisphere of the Globe in which only the *End of the Transit* can be seen.

To all the Countries that lie under the Meridian, *Venus* will appear to go off from the *Sun's* Disk at Noon. These are all the Eastern Parts of *Russia*, the *Caspian Sea*, the Western Part of the Kingdom of *Persia*, and the Eastern Parts of *Arabia*, with the North Part of the Island of *Madagascar*.

In the Western Semicircle of the Horizon you see those Countries where the End of the Transit will appear at Sun-rising, among which are the Eastern Parts of *Hudson's Bay*, in *North America*; *North Britain*, *Nova Scotia*; and a small Tract of the Eastern Parts of *Brazil*. In this Quarter of the Globe, among those who see the End of the Transit in the Forenoon, you observe the whole Continent of *Africa*, the greatest Part of *Europe*; our Island in particular, in North Latitude,

titude, and *St. Helena* in the South, viewing this Phænomenon nearly at the same Time of the Day.

In the Eastern Semicircle of the Horizon you see a few Countries lie; but most of them unknown to us; and probably not an Inhabitant among them all will see *Venus* make her Exit from the *Sun's* Disk at his setting. However, a great Part of the World, the vast Continent of *Asia*, will have the Pleasure of viewing this critical Phænomenon some Time in the Afternoon, if the Weather prove favourable to them.

Thus you proceed to find the various Appearances of the Transit for any other Place, as well as *London*. Having first observed the Difference of Longitude between the Meridian of *London* and the given Place, and converted it into Time, by which Means the Times of the Beginning, Middle, and End of the Transit become known, and the various Appearances represented for that Place, in the same Manner as we have now done for *London*.

The diurnal Motion of the Globe being in a Direction from West to East, and that of *Venus* over the *Sun's* Disk being the contrary Way, from East to West, will cause her to be seen a less Time on the Surface of the *Sun*, than she would have appeared if the Earth were at rest; or if she was viewed only from the Earth's Center, which is at rest with Respect to the superficial Parts; for in either of those Cases, we should see the true and real Motion of *Venus*. Whereas now, as we are moving towards the Planet, or to meet it, the Time of the Beginning or End of the Planet will necessarily be contracted. And this will be the Case of all the People that live in such Parts of the Earth where the Beginning and End can be observed, as at *Bencoolen*, in *Sumatra*; and all other Parts of the *East Indies*. But as every Place, while it describes the nocturnal Part of its Parallel, has a different Direction of Motion from that in the diurnal Part with Respect to a distant Point in the Heavens, so those Motions in the nocturnal Parts of the Parallels will have the same Direction as the Planet *Venus* itself; and consequently such as are in a Situation to view the Beginning and End of the Transit, while they describe their nocturnal Arches, will have the Duration of the Transit thereby prolonged; for if we move the same Way with another Body in Motion, that Body will appear to move more slowly than it would do if we ourselves had no Motion at all.

Now if we suppose the Planet *Venus* were to traverse the same Disk so near the *Sun's* Center as Dr. HALLEY, in his following Dissertation, has supposed, then her Path over the *Sun* would be considerably longer than Six Hours and a Quarter, before-mentioned, viz. almost Eight Hours; which would have given an Opportunity for observing the Beginning and End of the Transit by Persons moving in both the above-mentioned Directions, viz. Those of the *East Indies*, the contrary Way to the Planet, and those at *Fort Nelson* (at the Mouth of *York River*, at *Hudson's Bay*, in *North America*) moving the same Way with the Planet; for then those latter Observers at *Fort Nelson* would not only have observed the Beginning of the Transit at Sun-set, but likewise the End of it at Sun-rising; and thereby have observed a Duration of the Transit much greater than those in the *East Indies*, as you may easily see by rectifying the Globe, as before directed, for the Hours of II. in the Morning, and Three-quarters after IX. for the End.

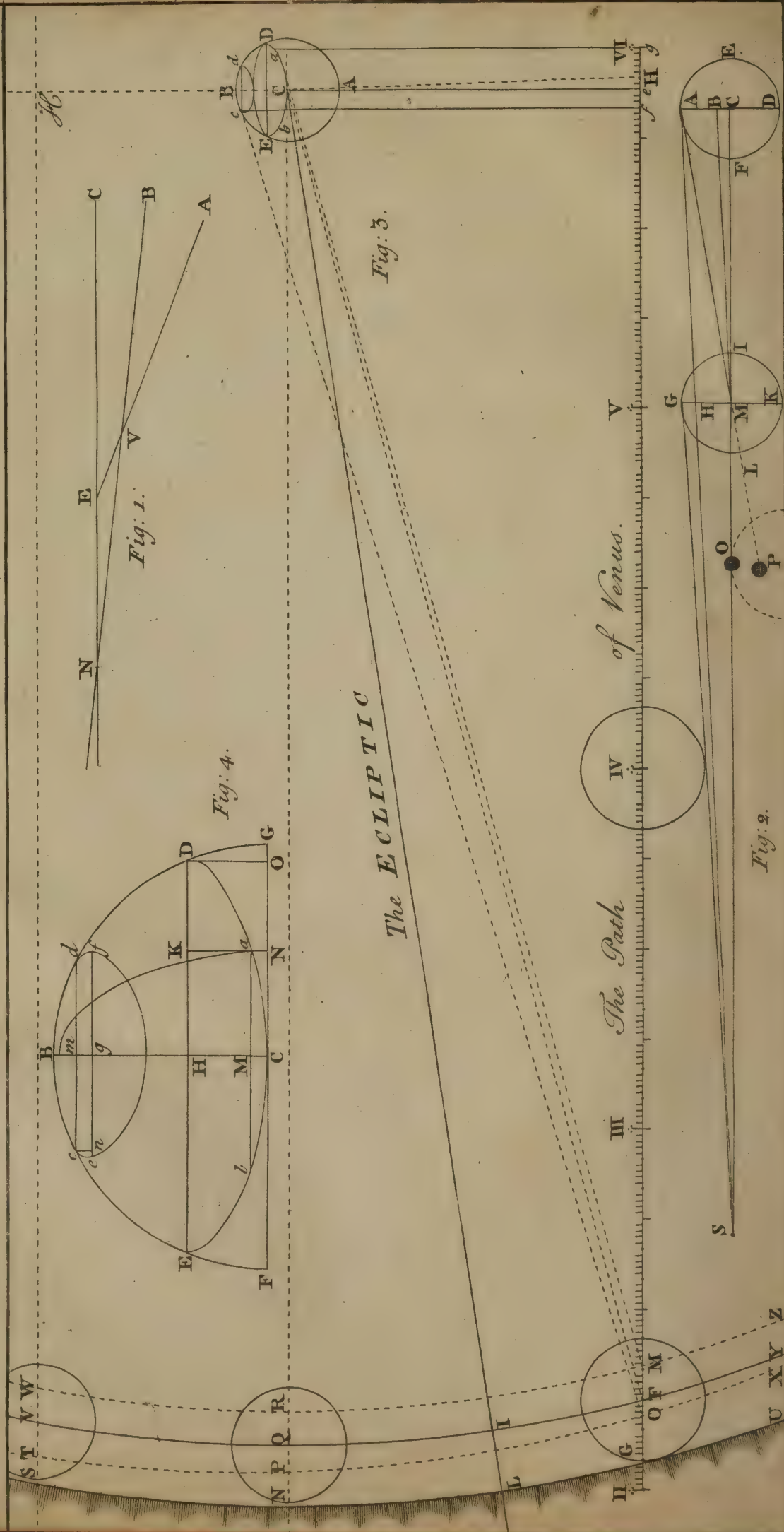
But there is great Reason to suppose (as we shall hereafter shew) that by reason of the Motion of *Venus's* Nodes, she will be carried much lower on the *Sun's* Surface, and thereby the Time, both at the Beginning and End of the Transit, will be contracted; and though not enough, at the Beginning of the Transit, to render it invisible at *Fort Nelson*, yet the End of it at Three-quarters after VIII. will be so much earlier than before, that *Fort Nelson* will not have reached the Western Horizon at the Time that *Venus* departs from the *Sun's* Disk at her Rising; all which is plainly observed from the Globe rectified as before-mentioned for those particular Times.

Since the Doctor's Dissertation is to shew how the *Parallax*, and consequently the *Distance*, of the *Sun* from the *Earth*, may in a great Degree be ascertained from the Difference between the true and apparent Duration of the Transit, it follows, that if the Beginning and End of the Transit could have been observed at *Fort Nelson* as well as at the *Indies*, that then this Discovery could have been made to that Degree of Exactness which the Doctor proposes; viz. to within at least a five hundredth Part of the whole. But as the Case is now like to prove, the apparent Duration by its Contraction only at the *Indies* will have a less Difference from the true Time, and therefore we must be content, at present, with a less perfect

fect Determination of the *Sun's* Parallax than we were at first in Expectation of.

But as another Transit of the same Planet, at the same Node, will happen in the Year 1769, on the 3d of *June*, which would have proved of little or no Service to us, had the present Transit been such as we first expected; yet, in Proportion as this is rendered more deficient for answering the proposed Discovery, that will be made more subservient to it; and, indeed, will fully answer all that can be expected from a Transit, and leave nothing more to be hoped for. So that, upon the Whole, we shall have two Transits, by this means, instead of one; and if we make the best Use of each, there is no doubt but Astronomy will, in ten Years Time, attain to its ultimate Perfection. But of this Subject, and more particularly of the Transit in 1769, we may take another Opportunity of discoursing more at large.

A PROJECTION and CALCULATION of the TRANSIT of VENUS.



Dr. HALLEY's
DISSERTATION

On the METHOD of Determining the
PARALLAX of the SUN

BY THE
TRANSIT of VENUS, JUNE 6. 1761.

N. B. *This Dissertation was delivered to the Royal Society, and published in their Transactions, N^o 348.*

I. **N**O Problem seems of a more difficult Nature than that which is proposed, to determine the Distance of the *Sun* from the *Earth* near the Truth; which yet, from proper Observations *, obtained at particular Times and Places, may be solved without much Labour; and what I now propose to this Society is, to shew our † young Astronomers (who may live to make those Observations) a Method ‡, by which they may measure the

* The Observations here referred to are those which are to be made on the Transit of *Venus* over the Face of the *Sun*, in the Year 1761, at the *East Indies* on one Part, and at *Hudson's Bay* on the other, as is more fully declared in the Sequel of this Dissertation.

† The Doctor had Reason to mention *young Astronomers*, because at the Time he wrote this, which was about the Year 1718, the Transit of *Venus* was at the Distance of 43 Years.

‡ The Method proposed by the Doctor for this Purpose is the Subject of this Dissertation, *viz*, The Difference of the Times of the Transit of *Venus* over the *Sun's* Disk, observed from the two distant Parts of the *Earth* before-mentioned. The Doctor, who is on all hands allowed to be the greatest Judge in those Things, has given very good
Reasons

the immense Distance of the *Sun*, within the 500th Part of the Whole.

II. It is well known that this Distance of the *Sun* from the *Earth* is supposed different by different Astronomers; *Ptolemy* and his Followers, as also *Copernicus* and *Tycho Brabe*, have computed it at 1200 Semi-diameters of the *Earth*; *Kepler* at almost 3500; *Riccioli* doubles this last Distance, and *Hevelius* makes it only half as much *.

III. But at length it was found, upon observing by the Telescope, *Venus* and *Mercury* on the *Sun's* Disk, divested of their borrowed Light, that the apparent Diameters of the Planets were much less than hitherto they had been supposed to be; and in particular, that

Reasons why the Planet *Mercury*, seen in the *Sun*, will not afford so good an Opportunity for determining the *Sun's* Parallax as the Planet *Venus*; concerning which, in a former Dissertation (see *Transf.* N^o 193.) as well as in the present, the Doctor speaks of the Transit of *Venus* as that by which alone this great Point can be obtained. This Sight, says he, which is by far the noblest that Astronomy affords, like the Secular Games, is denied to Mortals for a whole Century by the strict Laws of Motion; it will be afterwards shewn, that by this Observation alone the Distance of the *Sun* from the *Earth* may be determined with the greatest Certainty, which, on account of the Parallax otherwise intirely insensible, hath not hitherto been precisely defined; and at the Conclusion of that Number, he hath this remarkable Paragraph:

The principal Use of these Conjunctions is accurately to determine the Distance of the Sun from the Earth, or his Parallax, which Astronomers have by several Methods attempted in vain, whilst the Smallness of the Angles sought, does easily elude the nicest Instruments; but in observing the Ingress of Venus into, and Egress from, the Sun, the Space of Time between the Moments of the internal Contacts may be obtained to a Second of Time; that is, $\frac{1}{15}$ of a Second, or 4''' of the observed Arch, by Means of an ordinary Telescope, and Clock that goes accurately for six or eight Hours.

* The Methods by which the above-mentioned Astronomers attempted to determine the Parallax of the *Sun* may be seen at large in their Writings, or in the Astronomical Lectures of Mr. *Whiston*, Dr. *Keil*, and other modern Books of Astronomy. The most antient, and by far the most considerable Geometrical Method for determining the Parallax of the *Sun*, is that of the famous *Diagram* of *Hipparchus*, which was used by all the antient Astronomers. The second Method is, by the Observation of the *Luna Dichotoma*, or *Moon*, when she is in one of her Quarters, and appears to us exactly bisected. This Way was used by Astronomers of the last Age. The third Method is, by the Observation of the Parallax of *Mars*, invented by *Cassini*, and followed by *De la Hire*, *Flamsteed*, and others. But after all their most diligent Observations, and the Use of the most exquisite Instruments, they could arrive at no greater Precision than this, that the *Sun's* Parallax was certainly more than nine Seconds, but less than twelve; so that the Distance of the *Sun* could not be known within a fourth or fifth Part of the Whole; whereas by the Doctor's new Method it will be known to an hundred Times that Exactness, and without any great Nicety or Skill required in the Instruments or Observers.

Venus's

Venus's Semi-diameter, seen from the *Sun*, only subtends the fourth Part of a Minute, or fifteen Seconds: and that *Mercury's* Semi-diameter, at his mean Distance from the *Sun*, is seen under an Angle ten Seconds only, and *Saturn's* Semi-diameter under the same Angle; and that the Semi-diameter of *Jupiter*, the biggest of all the Planets, subtends no more than the third Part of a Minute at the *Sun*; whence, by Analogy, some modern Astronomers conclude, that the *Earth's* Semi-diameter, seen from the *Sun*, subtends a mean Angle between the greater of *Jupiter* and the lesser of *Saturn* and *Mercury*, and equal to that of *Venus*, viz. one of fifteen Seconds; and consequently that the Distance of the *Sun* from the *Earth* is almost 14000 Semi-diameters of the latter *.

IV. Another Consideration has made these Authors enlarge this Distance a little more; for since the *Moon's* Diameter is something more than a Quarter of the *Earth's* Diameter, if the *Sun's* Parallax be supposed fifteen Seconds, the Body of the *Moon* would be bigger than that of *Mercury*; to wit, a secondary Planet bigger than a primary one; which seems repugnant to the regular Proportion and Symmetry of the Mundane System.

V. But, on the contrary, it seems hardly consistent with the same Proportion, that *Venus*, an inferior Planet, and without any Satellite, should be bigger than our *Earth*, a superior Planet, and accompanied with so remarkable a Satellite; therefore, at a Mean, supposing the *Earth's* Semi-diameter seen from the *Sun*, or which

* Sir ISAAC NEWTON collected from Mr. Pound's Observations made with a Micrometer applied to the *Huygenian* Telescope of 123 Feet, that an Observer at the *Sun* would see *Saturn* at his mean Distance under an Angle of 16''; and *Jupiter* under an Angle of 37''. Mr. *Huygens* found the apparent Diameter of *Mars* not to exceed 11''. Dr. HALLEY himself observed the Diameter of *Mercury* to be 21'' $\frac{1}{2}$, and thence concludes that of *Venus* to be 30'' at her mean Distance (see *Philos. Transf.* N^o 386.) in all these Cases the Eye is supposed to be placed in the *Sun*.

Upon Supposition that the horizontal Parallax is 15 Seconds, the Distance of the *Sun* in Semi-diameters of the *Earth* is found by the following Analogy.

As the Tangent of	_____	15''	_____	5,861666
Is to Unity	_____	1	_____	0.
So is Radius	_____		_____	10,000000
To the Semi-diameters	13750 in the	}	_____	4,138334
Distance of the <i>Sun</i>				

Accordingly if the Parallax be 12'' $\frac{1}{2}$, the Distance of the *Sun* will come out nearly 16500 Semi-diameters of the *Earth*.

is the same thing, the *Sun's* horizontal Parallax, to be twelve Seconds and a half, the *Moon* will be less than *Mercury*, and the *Earth* bigger than *Venus*, and the *Sun's* Distance from the *Earth* come out nearly 16500 Semi-diameters of the *Earth* *.

VI. The Doctor admits of this Distance at present, till its precise Quantity be made to appear more certain by the Trial he proposes; nor does he regard the Authority of such as set the *Sun* at an immensely greater Distance, relying on the Observations of a

* A general Method for determining the Proportion of the Magnitude of the several Planets is easily deduced from the Ratio of their Distances from the *Sun*, which, according to the periodical Times of their Revolutions, are as follows:

♄	♀	⊕	♂	♃	♅
38710	72333	100000	152369	520096	954006

Therefore, let D = apparent Diameter of the *Sun*.

P = ditto of the *Earth*.

d = the apparent Diameter of the Planet.

a = Distance of the *Earth* from the *Sun*.

b = the proportional Distance of the Planet.

S = the real Diameter of the *Sun*.

E = ditto of the *Earth*.

N = ditto of a Planet.

Then it will be $D : P :: S : E$; and because $d \times \frac{b}{a}$, is the apparent Diameter of the Planet

seen from the *Sun* at the Distance of the *Earth*, therefore $D : d \times \frac{b}{a} :: S : N$. Consequently

$E : P :: N d \times \frac{b}{a}$; or $E : N :: a P : d b$; whence it appears, that the true Proportion of

the Magnitudes of the Planet and *Earth* will be known when the *Sun's* Parallax is determined; and that the Diameter of the Planet is reciprocally proportional to that Parallax, or half the Quantity P .

Upon Supposition therefore that the *Earth's* apparent Diameter is $30''$, and that of *Mercury* $20''$, then the Proportion of their Magnitude will be as 100000×30 to 38710×20 ; that is, as 300000 to 77420, or as 109 to 28 nearly; whereas the Magnitude of the *Earth* is to that of the *Moon* as 109 to 30; therefore the Supposition makes *Mercury* less than the *Moon* in the Ratio of 28 to 30, or of 14 to 15. But if the *Sun's* Parallax be $12'' \frac{1}{2}$, or the apparent Diameter of the *Earth* $25''$, then 100000×25 is to 38710×20 as 109 is to 34 nearly; in this Case, therefore, the Planet ♄ will be bigger than the *Moon*, in the Proportion of 34 to 30. And the *Sun's* Distance will be nearly 16500 Semi-diameters of the *Earth*. On the other hand, the *Sun's* Parallax being $15''$, that of *Venus* will be $54''$ (see the Note to Art. VII.), and the Magnitude of the *Earth* and *Venus* will be as 100000×30 to $72333 \times 54''$, or as 30 to 39, nearly; so that *Venus* would, in this case, be near $\frac{1}{3}$ Part bigger than the *Earth*.

vibrating

vibrating *Pendulum*, which are not, as it seems, accurate enough to determine such minute Angles; at least such as use this Method will find the Parallax sometimes none at all, and sometimes even negative; that is, the Distance will become either infinite, or more than infinite, which is absurd; and it is scarce possible for any one certainly to determine, by Means of Instruments, however nice, single Seconds, or even ten Seconds; and therefore it is not at all surprising that the exceeding Minuteness of such Angles. has hitherto baffled the many and ingenious Attempts of Artists*.

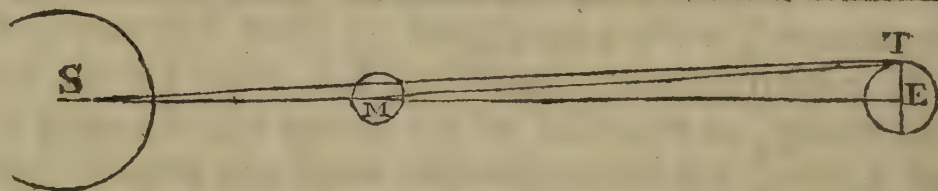
VII. As the Doctor was making his Observations in the Island of *St. Helena*, about forty Years before, on the Stars round the South Pole, he happened to observe with the utmost Care *Mercury* passing under the *Sun's* Disk; and, contrary to his Expectation, he very accurately obtained, with a very good 24 Foot Telescope, the very Moment in which *Mercury*, entering the *Sun's* Limb, seemed to touch it internally, as also that of his going off, forming an Angle of internal Contact; whence he discovered the precise Quantity of Time the whole Body of *Mercury* had then appeared within the *Sun's* Disk, and that without an Error of one single Second of Time; for the Thread of Solar Light, intercepted between the obscure Limb of the Planet, and bright Limb of the *Sun*, though exceeding slender, affected his Sight, and in the Twinkling of an Eye both the Indenture made on the *Sun's* Limb by *Mercury* entering into it vanished, and that made by his going off appeared. Upon observing this, he immediately concluded, that the *Sun's* Parallax might be duly determined by such Observations, if *Mercury*, being nearer the *Earth*, had a greater Parallax when seen from the *Sun*; for this Difference of Parallaxes is so very inconsiderable, as to be always less than the *Sun's* Parallax, which is sought; consequently, though *Mercury* is to be

* The above Paragraph mentions the Vibrations of a Pendulum as the Means of determining the *Sun's* Distance; but I find no mention made of this Method by any other Author; and from the Nature of the Thing, it must appear precarious, uncertain, and insufficient for any such Purpose.

frequently

frequently seen within the *Sun's* Disk, he will scarcely be fit for the present Purpose *.

VIII. Therefore, there remains *Venus's* Transit over the *Sun's* Disk, whose Parallax, being almost four Times greater than that of the *Sun* †, will cause very sensible Differences between the Times in which *Venus* shall seem to pass over the *Sun's* Disk in different Parts of our *Earth*: From these Differences, duly observed, the Dr. affirms, the *Sun's* Parallax may be determined, even to a small Part of a Second; and that without any other Instruments than Telescopes ‡ and good common Clocks, and without any other Qualifications in the Observer than Fidelity and Diligence, with a little Skill in Astronomy; for you need not be scrupulous in finding



* The *Sun's* Parallax having been stated at $12'' \frac{1}{2}$, the Parallax of *Mercury* is easily known by the known Ratio of its Distance from the *Sun* and *Earth*; but this will be best explained by a Diagram: Therefore let S be the *Sun*, M *Mercury*, and E the *Earth*, whose Semi-diameter is E T; draw the Lines S E, S T, and M T, then is the Angle T S E = $12'' \frac{1}{2}$ the *Sun's* Parallax, and E M T is the Parallax of *Mercury*, and the Difference of those Parallaxes is the Angle M T S. Now as these Parallaxes, and their Difference, are proportional to the Lines M T, S T, and S M, which are in a given Ratio to each other; they will also have a constant Proportion among themselves, and therefore when any one of them is known, or supposed to be known, the others will be known according to that Hypothesis.

For Example; if the Solar Parallax T S E be $12'' \frac{1}{2}$, then it will be M T : S T (or M E : S E) :: 6129 : 10000 :: $12'' \frac{1}{2}$: $20'' \frac{3}{8}$ = T M E, the Parallax of *Mercury*, nearly; then the Difference is $7'' \frac{4}{5}$ for the Angle M T S, on the above Supposition. Now this Difference of Parallaxes is less than the *Sun's* by more than $4'' \frac{1}{2}$; and since the Parallax of the *Sun* is to be found by the Difference of the Parallaxes from Observation, the Planet *Mercury* will not be so proper for this Purpose as *Venus*, where the Difference of Parallaxes will be so much more considerable.

† By the foregoing Proportions of the planetary Distances from the *Sun*, the Distance of *Venus* from the *Earth* is to the *Sun's* Distance as 27668 to 100000; therefore say, as 27668 : 100000 :: $12'' \frac{1}{2}$: $45''$ nearly, which is the Parallax of *Venus*, from which take the *Sun's* Parallax $12'' \frac{1}{2}$, there remains $32'' \frac{1}{2}$ for the Difference, or Parallax of *Venus* from the *Sun*, which is more than four Times as great as that of *Mercury*, $7'' \frac{4}{5}$, found in the last Note. Also *Venus's* Parallax $45''$ is almost four Times larger than ($12'' \frac{1}{2}$) the Parallax of the *Sun*.

‡ If the *Sun* be viewed with a Telescope which magnifies about 15 or 20 Times, it will suffice for shewing the Contacts of the Limbs of the solar and venereal Disks; and also if the Micrometer be adapted in the Focus of the Eye-glass, the Latitude of *Venus* in the Middle of the Transit may be measured, and thereby her Theory, Motion of her Nodes, &c. may with great Accuracy be ascertained.

the

the Latitude of the Place, or in accurately determining the Hours with respect to the Meridian; it is sufficient if the Times be reckoned by Clocks, truly corrected according to the Revolutions of the Heavens †, from the total Ingress of *Venus* below the *Sun's* Disk, to the Beginning of her Egress therefrom; when her opaque Globe begins to touch the bright Limb of the *Sun*; which Times, as the Dr. found by Experience, may be observed even to a single Second of Time.

IX. But, by the limited Laws of Motion ||, *Venus* is very

† The Clocks or Watches used in observing this Phænomenon may be easily corrected, by observing the Revolution of a fixed Star to a given Object for a few Nights before-hand, in the following Manner.—Let a Telescope, with cross Hairs in the Focus of the Eye-glass, be fixed in some convenient Place to observe the Transit of a Star over the vertical Hair; and there let it remain immoveable for some Time. Then observe the Moments of Time when the Star comes upon the vertical Hair two Nights successively, and if the Time of this Revolution of the Watch or Clock, by which you observed it, be just 23 Hours, 56 Minutes, and 4 Seconds, you will be satisfied it goes true. But otherwise you must alter it, till you find it will shew the Return of the Star to the Hair sooner by three Minutes and 56" each successive Night, than the foregoing. For want of a Telescope, a small Hole made in a Window-shut, to observe the Disappearing of a Star behind a Chimney, or any opaque Object that is sharp, will do nearly as well.—The Reason of such Correction is, because 3' : 56" of Time, answer to 59' : 8" of Motion, which is that by which the Motion of the *Earth* in 24 Hours exceeds one entire Revolution on its Axis, which is always made in 23^h : 56' : 4" precisely.

The Reason why no great Accuracy in regard to the Latitude of the Place is necessary, will appear hereafter. Nor is it requisite to define the Hour of the Day for the several Phænomena of the Transit; the Discovery of the solar Parallax depending only upon the true Quantities of the Duration thereof, and their Differences, as observed in two distant Places, on opposite Parts of the Meridian.

|| In Trans. N°. 139 here referred to, the Dr. has shewn from the Theory of *Venus's* Motions, the Method of ascertaining the Times when she will transit the *Sun's* Disk, both at the ascending and descending Nodes, and tabulated them as below.

In the Month of NOVEMBER.

Year	Time of Conj.			Dist. from Cent.		
	d.	h.	'	'	"	
918	20	21	53	6	12	B
1161	20	21	10	6	55½	A
1396	23	7	20	4	38	B
1631	26	17	29	16	11	B
1639	24	6	37	8	30	A
1874	26	16	46	3	3	B
2109	29	2	56	14	36	B
2117	26	16	3	10	5	A

In the Month of MAY.

Year	Time of Conj.			Dist. from Cent.		
	d.	h.	'	'	"	
1048	24	13	45	3	50	B
1283	23	8	14	5	31	A
1291	25	15	9	14	27	B
1518	25	16	32	14	52	A
1526	23	9	37	5	6	B
1761	25	17	55	4	15	A
1769	23	11	00	15	43	B
1996	28	2	13	13	36	A
2004	25	19	18	6	22	B

N. B. In the Tables A denotes the Transit over the southern Part of the solar Disk, and B over the northern Part. And by adding 11 Days to the *Old Stile* of the Tables, they are adapted to the *New*.

rarely

rarely seen within the *Sun's* Disk; and for a Series of 120 Years and upwards is not to be seen there once; that is, from 1639, when Mr. *Horrox* was favoured with this agreeable Sight, and he the first and only one since the Creation of the World, down to 1761 §; at which Time, according to the Theories hitherto found agreeable to the Heavens, *Venus* will pass under the *Sun* on May 26 in the Morning*; so that (vide *Phil. Transf.* N^o. 193.) at *London*, almost at six o'Clock in the Morning, she is to be in the Middle of the *Sun's* Disk, and but four Minutes more southerly than his Center; the Duration of this Transit will be almost 8 Hours; that is, from two till almost ten o'Clock in the Morning, and consequently her Ingress will not be visible in *England*; for the *Sun* at that Time being in 16° of *Gemini*, and almost in 23° of north Declination, shall be seen not to set throughout the whole northern frigid Zone; and consequently the Inhabitants of the Coast of *Norway*, as far as its northern Promontory beyond the Town of *Drontheim*, may observe *Venus* entering the *Sun's* Disk, and perhaps this Ingress into the *Sun* at his Rising, may be seen by the Inhabitants of the North of *Scotland* and those of *Zetland*; but when *Venus* is nearest the

§ Mr. *Horrox* (whose Life is in Page 271 of our *BIOGRAPHIA PHILOS.*) from his Skill in Astronomy was able to predict the Transit of *Venus* in 1639; and having acquainted Mr. *Crabtree* therewith in a Letter dated *October* 26, 1639, he desired him to observe it with all Attention possible, and particularly to measure the Diameter of *Venus*, which, says he, *Kepler* has made 7', *Lansbergius* 11', but I can make it no more than 1'.

Accordingly when the Day came, he made his Room dark, and took in the *Sun's* Disk through a Telescope on a Paper Screen, and there observed the Transit, and found *Venus's* Diameter to be nearly 1' : 10". After this (on *April* 20, 1640) he wrote to Mr. *Crabtree*, to inform him a second Time, how much he had made the Diameter of the Planet in his Observation; for, says he, "That I have forgot, but I remember what else you observed very well." Hence it appears that this Transit was observed by Mr. *Crabtree* as well as by himself, and that they mutually compared their Observations on the Subject. Of this Phænomenon Mr. *Horrox* gave a large Account in a Tract he published, entitled, *Venus in Sole visa*, which 22 Years after was published by *Hevelius* with his *Mercurius in Sole visus*.

* According to the Theories, on which the Tables published by *Cassini*, *Flamsteed*, and *Street*, were constructed, the Calculation of the preceding Transit (in 1639) answered very well to Observation. For by Mr. *Horrox*, the geocentric Longitude of *Venus* was observed to be 7s : 14° : 26' : 30" at the Time of the total Ingress, and the *Caroline* Tables give but 1" more. Again the observed geocentric Latitude at the same Moment was 10' : 27", and by the Tables it was 10' : 30". Since then, the Theory of *Venus* answered so nicely to Observation at that Time, one might expect it should

the *Sun's* Center, the *Sun* will be vertical to the northern Coasts of the Gulf of *Ganga*, or rather of the Kingdom of *Pegu*; and consequently

should do so now; and therefore I shall give the Reader a Summary of the Calculation for the approaching Transit, which I have made from the same Tables.

The mean Motion of VENUS.				°	'	''	'''
For	A. D. 1761	—	—	2	16	34	55
	May	—	—	6	12	15	20
	Days 25	—	—	1	10	3	12
	Hours 17	—	—	0	1	8	5
	Minutes 55'	—	—	0	0	3	4
The mean Anomaly of Venus				10	10	5	12
The heliocentric Longitude of Ditto				7	15	42	20
Precession of the Equinox add					29	46	16
Longitude from the Equinox				8	15	28	36
Place of the Earth				8	15	35	37
Difference				0	0	7	1
Place of the Node				8	15	2	0
Distance of Venus from ☿				0	0	26	36
Distance of Earth from ☿				0	0	33	57
Inclination of South				0	0	1	34
Curtate Distance of Venus							
Distance of the Earth							
As the Sum				174225		5,241099	
To the Difference				28889		4.460732	
So is the Tangent of $\frac{1}{2}$ Sum of				89° : 56' : 30''		12.996683	
To Tangent of $\frac{1}{2}$ the Difference				89 : 39 : 6		12.216316	
The Parallax of the Orbit				179 : 35 : 36			
The Elongation of Venus				0 : 17 : 24.			
Then, as the Sine of Commutation				7'		0,845098	
To the Sine of Elongation				17,4		1,240549	
So is the Tangent of Inclination				1' 34''		6,658703	
						7,899252	
To the Tangent of the Latitude of Venus, as seen from the Earth				3' 54''		7,054154.	
				C		Thus	

frequently in the neighbouring Countries, when the *Sun* shall, at the Ingress of *Venus*, be almost four Hours distant to the East, and almost as many to the West at her Egress, her apparent Motion within the *Sun's* Disk will be accelerated almost twice as much as is the horizontal Parallax of *Venus* from the *Sun*; because *Venus* at that Time moves retrograde from East to West; whilst in the mean Time an Eye, on the Surface of the Earth, is carried the contrary Way from West to East.

Thus it appears, that at the Middle of the Transit *Venus* will be distant from the *Sun's* Center $3' : 54''$; and because the Dr. makes it but $4'$, it is plain he computed by the same Theory or Tables. But by his own Tables lately published, this Latitude of *Venus* is made $9' : 51''$, as appears by Mr. *Metcalf's* Computation in the following Table.

6 June 1761, Apparent Time A. M.				h.	m.	s.
First Contact	---	---	---	2	5	23
Central Ingress	---	---	---	2	16	41
Total Immersion	--	---	--	2	28	15
Middle of the Transit	--	--	--	5	24	17
Nearest Approximation of Centers			---	5	24	50
Ecliptic Conjunction	--	--	--	5	46	$17\frac{1}{2}$
Emerfion	--	---	---	8	20	$19\frac{1}{2}$
Central Egress	--	--	--	8	31	54
End of the Transit	--	--	--	8	43	12
Central Duration	--	--	--	6	15	12
Duration of the Transit		--	--	6	37	49
Lat. of ♀ at the ecliptic Conjunction			--	9'	51''	16'''
Horary Motion of <i>Venus</i> from the <i>Sun</i>			--	3	56	57

A *French* Author in his *Ephemerides* has also made the Latitude and other Phænomena different from either of the foregoing. The Particulars whereof for the Meridian of *Paris*, are as follow.

		h.	'
The Ingress begins	--	13	47
is Central	--	13	58
Total	--	14	10
Middle of the Transit	--	17	14
Egress begins	--	20	9
Central	--	20	26
Total	--	20	42
Duration of the Transit	--	6	55
Latitude at the Conjunction	--	8'	57''

Besides the Parallax, another great Use of the Transit will be to determine which of those differing Theories, or Sets of Tables, are nearest the Truth.

X. Sup-

X. Supposing the *Sun's* Parallax, as was said, to be twelve Seconds and a Half, *Venus's* Parallax will be forty-three Seconds; and subtracting the *Sun's* Parallax, there will remain half a Minute at least for the horizontal Parallax of *Venus* from the *Sun*, and consequently *Venus's* Motion will be accelerated $\frac{3}{4}$ of a Minute at least from that Parallax, whilst she passes over the *Sun's* Disk, in such Elevations of the Pole as are near the *Tropic*; and still more so in the Neighbourhood of the Equator; for *Venus* will at that Time accurately enough describe within the *Sun's* Disk four Minutes an Hour, and consequently at least eleven Minutes of Time (by which the Duration of this Eclipse of *Venus* will be contracted by Reason of the Parallax) answer to $\frac{3}{4}$ of a Minute; and by this Contraction alone we might safely determine the Parallax, provided the *Sun's* Diameter and *Venus's* Latitude were very accurately given, which yet we cannot possibly bring to a Calculation, in a Matter of such great Subtilty *.

XI. There-

* What relates to the *Geographical Phenomena* of the Transit, we have shewn in the INTRODUCTION, by the *terrestrial Globe*, which is infinitely preferable to any *Maps, Planispheres, or Projections* of any Kind for this Purpose; for they rather puzzle and perplex the ungeometrical Reader; but the *Globe* makes every Circumstance natural and easy.

In the Figure to Article VII, T S is a Line drawn from the Spectator's Eye at T on the *Earth's* Surface, to the Center of the *Sun*; and T M another Line drawn from the Eye to the Center of a Planet M; the Angle S T M which they contain, is called the *Parallax of the Planet from the Sun*, or the Difference of Places in which the Centers of the *Sun* and Planet appear from the Surface of the *Earth* at T, while at the same Time they appear from the Center E in the same Point or Line E M S.

This Angle S T M = T M E — T S M, or it is equal to the Difference between the horizontal Parallax of the Planet and *Sun*; which, in the Case of *Venus*, is 32''; but if we make the least of it, and take the Doctor's Number 30'' for the Parallax of *Venus* from the *Sun*; then since *Venus's* Motion upon the *Sun's* Disk is 4' of a Degree per Hour (as appears from her Theory); therefore if we say, as 240'' of Motion : 60' of Time :: 30'' of Motion : 45'' of Time, which is $\frac{3}{4}$ of a Minute, by which the Duration of her Transit is hourly contracted. But since the Transit is contracted at each End equally by the Parallax, the whole Contraction will be in Proportion to double that Parallax, or to 60'' of Motion, which is 1' 30'' per Hour in Time. And since the whole Motion of *Venus* over the Disk as seen from the *Earth's* Center, is performed in about 7 Hours and 20' (according to the Doctor's Hypothesis) therefore $7\frac{2}{3} \times 1\frac{1}{2} = 11''$, the whole Contraction; so that to a Person at *Bengal*, the Duration of the Transit will be but 7 Hours and 9 Minutes.

Now since this Motion of *Venus* is estimated in Minutes of a Degree on the *Sun's* Disk, therefore if the *Sun's* Diameter, and *Venus's* Latitude, or Distance from the Center of the *Sun* at the Middle of the Transit could be accurately known, then from

XI. Therefore we must have another Observation, if possible, in Places where *Venus* possesses the Middle of the *Sun* at Midnight, under the opposite Meridian; that is, 6^h or 90° more westerly than *London*, and where *Venus* enters the *Sun*'s Disk a little before his Setting, and goes off a little after his Rising; which will happen in the said Meridian in about 56° of N. Lat. that is, at *Nelson's Harbour* in *Hudson's Bay*; for in the neighbouring Places *Venus*'s Parallax will protract the Duration of the Transit, and make it at least six Minutes longer; because whilst the *Sun* seems to tend under the Pole from West to East, these Places on the *Earth*'s Surface will seem to be carried with a contrary Motion towards the West; that is, with a Motion conspiring with the proper Motion of *Venus*; consequently *Venus* will seem to move slower within the *Sun*'s Disk, and continue longer thereon †.

XII. If

a single Observation made in Parts near the Equator, the Parallax of *Venus* from the *Sun*, and from thence the *Sun*'s Parallax itself, might safely be determined. But since the Latitude of *Venus* is at present uncertain from Theory, and can only be determined at the Time of the Transit, we have no Way left but to make as great a Difference as we possibly can between the Durations of the Transit, as observed at two different Places. As the Parallax of *Venus* from the *Sun* will be the Cause of that Difference, it is plain the more we can contrive to magnify the Effect, the more completely we shall be able to discover and estimate the Cause. And for this Purpose, another Part of the *Earth* must be chosen, in a very different Parallel, and on the opposite Meridian, as will be shewn in the next Note.

† That the Difference of the Durations of the Transit may be as great as possible, the two Observations are to be made in Places where the Beginning and End can both be observed, and where the Velocity of *Venus*'s Motion will be as different as such a Condition will admit of; the Place therefore to which we are delegated for a second Observation, is *Port Nelson*, at the Mouth of *York River* in *Hudson's Bay*, in the Parallel of 56° Lat. The geographical Reasons for all which, are explained by the terrestrial Globe, in the Introduction.

As at *Bengal* the Transit is contracted, by Reason the Spectator's Eye is there carried in a Direction contrary to that of *Venus*'s Motion; so on the opposite Meridian, at *Port Nelson*, the Motion of the Spectator will conspire with that of *Venus*, and by that Means protract the Continuance of the Transit, as is evident from the Principles of Mechanics.

But the Quantity by which the Duration of the Transit is contracted in one Case, and protracted in the other, is proportioned to the Chord of the Arch through which the Spectator is carried by the *Earth*'s diurnal Motion from the Beginning to the End of the said Transit; these Chords are, in the Parallel of the *Ganges*, $22^\circ 30'$, and of *Port Nelson* 56° , as the Numbers 148 to 92 (as will appear hereafter) therefore say, as $148 : 92 :: 11' : 6\frac{1}{2}'$. Wherefore at *Hudson's Bay* the Transit will be prolonged $6' 50''$, or its Continuance will be $7^h 26' 50''$.

That

XII. If therefore, in both Places this Transit happen to be duly observed by proper Persons, it is evident that the *Mora* will be longer

P. I. That the young Astronomer (for whom this Comment is intended) may more readily see the Reason of the whole Affair, as far as Calculation is concerned in it, I shall lay before him a Specimen of a more accurate Computation for both Latitudes, which he will see illustrated by the Diagram in the Introduction, where *A F E G* is the *Earth's* Equator; *G H*, the Orbit of *Venus*; *A B C*, the *Sun*; *Y D Z*, the Path of *Venus* of the solar Disk. When therefore *Venus* just touches the *Sun's* Limb internally, it will appear to an Eye at *C*, the *Earth's* Center, in the Line *C Y*, and the Place in her Orbit is at *S*; but when she touches the western Limb internally at *Z*, then the Eye at *C* views her by the Ray *C Z*, and in the Orbit at *V*. So that *S V* is the Arch of angular Motion described in $7^h : 20'$ of Time, and is 110 Degrees, in the Equator = *K L*. *Z. I.*

Let *B I D H* be the Parallax^{*e*} of $22^\circ : 30'$; and *a b* the Chord of the Arch *a I b*; thro' which the Eye is moved by the diurnal Rotation of the *Earth*, in $7^h : 9'$, from *b* to *a*, which is 107° . Draw through *S* the Ray *b S w*, also the Ray *b Y*; then will the Angle *w b Y* (the Parallax of *Venus* from the *Sun*), shew her at a Distance from the *Sun* to an Eye at *b* in *Bengal*, when she is totally within the Disk, viewed from the Center *C*. But when the Planet arrives at *T*, it is evident, the Ingress appears compleat to the Eye at *b*. Therefore it will happen so much sooner at *b* than at *C*, as is equal to the Time spent in describing the Arch *S T*. *later*

Again, let *c f d* be the Parallel of 56° , and let the Eye of the Spectator at *Port Nelson* be at *c*, to view the entire Ingress by the Ray *c R Y*; then it is evident the Planet at *R* will be there seen in the *Sun* sooner than at the Center *C*, by the Time the Planet takes in describing the Arch *R S*. Note, *c d* is the Chord of the Arch *c d* of the Parallel through which the Eye is carried in the Time of $7^h : 26' \frac{5}{8}$; and is equal to $111^\circ : 45'$ of diurnal Motion.

Therefore the Difference between the Moments of the entire Ingress as viewed at the *Ganges* (*b*) and at *Port Nelson* under the opposite Meridian at (*c*) will be the Time in which *Venus* will pass over the Arch *R T*. The whole Difference therefore of the two Durations will be double that Time, since the Parallax gives the same Difference of Time for the Egress as for the Ingress, which Difference therefore is $17' : 50''$.

Since the middle Moment of the Transit is nearly the same to an Eye at the *Earth's* Center *C*, and to a Spectator at *I*, it is evident, the Difference which is made in the Time of the first half Duration at those Points *C* and *I*, is occasioned by the Motion of the Eye through the Arch *b I*, which is the very same as would happen if the Eye were carried in the right Line, or Chord *a b*, from *b* to *e*. And therefore that Difference will be as the Sine *b e* of half the Arch *b I a* = 107 , or the Sine of $53^\circ : 30'$.

In like Manner it is shewn, that the Difference of the half Duration of the Transit at *C* and *c*, is proportioned to the right Line *c g*, or Sine of (half *c d*) $55^\circ : 30'$ at least. Therefore the whole Difference of the Semi-durations at *Bengal* and *Port Nelson*, will be as the Sum of those Lines, or Sines, viz. as *b e* + *c g*.

If therefore in the *Earth's* Semi-diameter *C A* continued out, we take *C M* = *b e* + *c g*, and draw the Line *M Y*, to cut the Orbit of *Venus* in *Q*, and join *Q C*; then it is plain, there will happen the same Difference in the Time of the Beginning of the Transit to an Eye placed at *C*, and another at *M*, as there will be to the two Spectators at

longer by 17 entire Minutes in *Nelson's* Harbour than in the *East Indies*; nor does it matter much whether the Observation be made at *Fort St. George*, commonly called *Madras*, or at *Bencoolen* on the western Coast of the Island of *Sumatra*, near the Equator; but if the *French* should incline to make the Observation, *Pondecherry* on the western Coast of the Gulf of *Ganga*, at the Elevation of 12° will be a proper Place for the Purpose; and for the *Dutch*, *Batavia* their famous Emporium, is a fit Place: And, indeed, the Dr. would have several Observations made of the same Phænomenon in different Parts, both for further Confirmation, and lest a single Observer should happen to be disappointed by the Intervention of Clouds from seeing what the Dr. does not know if those either of the present or following Age shall ever see again; and upon which the certain and adequate Solution of the noblest, and otherwise most difficult Problem depends; therefore the Dr. again and again recommends it to the Curious strenuously to apply themselves to this Observation.

XIII. By this Means the *Sun's* Parallax may be discovered to within its five hundredth Part, which doubtless will seem surprising to

at (b) and (c). Also that there will be the same parallatic Angle $S C Q$ in each Case, or the Arch $Q S = (S T + S R) = R T$.

Then because the *Sun's* Diameter $A E = 31'$, the Angle $Y C Z$ will be $= 29' : 30''$ nearly; and therefore as the Arch $S V$, which measures that Angle in the Orbit of *Venus* is passed over in $7^h 20' = 440' 5$ and an Arch $= 2 Q S$ in $17' : 50''$; if we say as $26400'' : 1076'' :: 29' 30'' = 1770'' : 72''$; the half of which is $36'' = S C Q$, the Parallax of *Venus* from the Sun for the Base or Distance $C M$.

If the *Earth's* Semi-diameter $A C = 10000$, then the Sine of $53^\circ : 30' = b e = 7426$; and the Sine of $55^\circ : 30' = c g = 4629$; whence their Sum is $C M = 12055$. We have also shewn (Note to Article IX) that the Distance of the *Earth* from the *Sun* $C Y = 10155,7$ and the Distance of *Venus* $Q Y = 7266,8$; from whence the Distance of the *Earth* from *Venus* will be found $2923,4 = C Q$. Therefore say, as $Q Y = 726 : 36'' = S C Q :: Q C = 292 : 14'' 30''' = C Y Q$ or $C Y M$, which is the Parallax of the *Sun* for the Distance $C M$. And therefore also the Parallax of *Venus* $M Q C = 50'' : 36'''$.

But lastly, to adapt these horizontal Parallaxes of the *Sun* and *Venus* to the *Earth*; we must say, as $C M = 12 : A C = 10 :: 50'' 36''' : 46'' 34''' = A Q C$, the horizontal Parallax of *Venus*. And $12 : 10 :: 14'' 30''' : 12'' 5''' = A Y C$, the horizontal Parallax of the *Sun*, on the Supposition that the different Duration of the Transit in the Latitudes of $22^\circ 30'$ and 56° be observed to be $17' 50''$.

And if the Observation be made at *Fort St. George*, *Bencoolen*, *Pondecherry*, *Batavia*, or other Places near the Equator, it is evident the Chord $a b$ will be greater, and the Difference of the Durations will still be longer, and therefore give the *Sun's* horizontal Parallax in Proportion more accurately.

some;

some; yet notwithstanding, if an accurate Observation be had in both Places abovementioned, it has already been shewn, that the Duration of these Eclipses of *Venus* differ from each other by 17 entire Minutes, upon the Supposition that the *Sun's* Parallax is 12 Seconds and a Half; and if this Difference be found to be greater or less by Observation, the *Sun's* Parallax will be greater or less almost in the same Ratio; and since 17 Minutes of Time answer to 12 Seconds and a Half of the *Sun's* Parallax, for each Second of the Parallax there will arise a Difference of upwards of 80 Seconds of Time; therefore if this Difference be had true within two Seconds of Time, the Quantity of the *Sun's* Parallax will be had to within the fortieth Part of one Second; and consequently his Distance will be determined to within its five hundredth Part at least, if the Parallax be not found less than what the Dr. supposes it; for, 40×12 and $\frac{1}{2}$ is 500 *.

XIV. Here the Dr. has had no Regard to the Planet's Latitude, both to avoid the Trouble of a more intricate Calculation, which would render the Conclusion less evident, as also on Account of the Motion of the Nodes of *Venus* not being hitherto discovered, and which cannot be duly determined but by such Conjunctions of the Planet with the *Sun* as this; for it was only on the Supposition that the Plane of *Venus's* Orbit is immoveable in the Sphere of the

* It appears by the last Note, that if the Difference of the Duration of the Transit at the *East Indies* and *Hudson's Bay* be $17' : 50'' = 1070''$, then the *Sun's* horizontal Parallax will be $12''$ at least; therefore $1070'' : 12'' :: 1'' : \frac{12}{1070} = \frac{1}{90\frac{1}{2}}$ of $12''$; or each Second of the Difference in Duration will correspond to the 1070th Part of the whole Solar Parallax. Therefore the Distance of the *Sun*, which is proportional to the Parallax, will, in this Case, be discovered to be within at least a 1000th Part of the Whole.

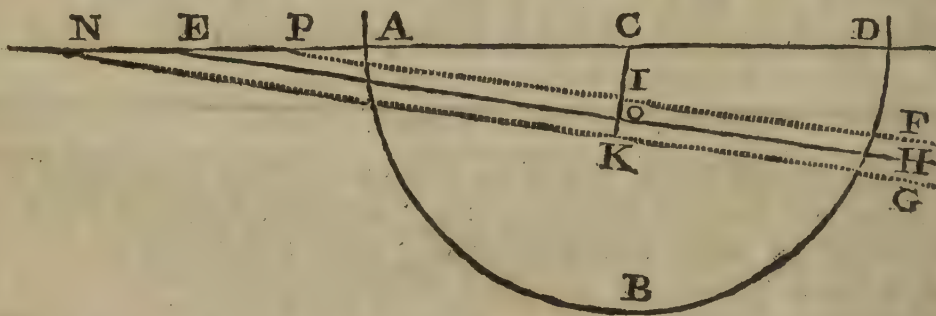
But if the Nodes of *Venus's* Orbit have a retrograde Motion, and so occasion her to pass over the Disk with a much more southern Latitude, according to the Doctor's own Tables, the *Mora* will be less, as also the Difference of the Transit's Duration at the above Places; but though it should be found to be but one half the Quantity we have now supposed it, yet even then it will give the *Sun's* Parallax and Distance to within a 500th Part of the Whole; which is more than 100 Times the Accuracy our best Observers could ever yet pretend to; for they have only discovered that the *Sun's* Parallax is more than $9''$, but less than $12''$, and so leave us in doubt about a fourth Part of the Whole; which, according to our present Reckoning, is not less than (20000000, or twenty Millions of Miles. And indeed if we make the best use possible of this most favourable Opportunity, we must after all sit down with the disagreeable Assurance, that we can never arrive to the Knowledge of the *Sun's* true Distance within 80 or 100 thousand Miles.

fixed

fixed Stars, and that her Nodes would continue in the same Places wherein they were in 1639, that it was concluded that *Venus* would pass four Minutes below the *Sun's* Center: And if in 1761 she should pass more southerly, it will be evident that there is a Regression of the Nodes; but if more northerly, that there is a Progression of them; and that at the Rate of five Minutes and $\frac{1}{2}$ in 100 *Julian* Years, for each Minute, by which the Path of *Venus* will at that Time be more or less distant from the *Sun's* Center than the said four Minutes; but the Difference between the Durations of these Eclipses will be somewhat less than 17 Minutes, by Reason of the southern Latitude of *Venus*; but greater, if, by the Progression of the Nodes, she shall pass over the *Sun* to the North of his Center †.

XV. But

† What the Latitude or Distance of *Venus* will be from the Center of the *Sun* at the true Conjunction cannot be known, till it be determined at what Rate the Nodes of *Venus's* Orbit do move; which may be easily discovered at the Time of the Transit, but hardly by any other Means. But in order to elucidate what the Doctor says farther in this Passage, we add the following Diagram:



Let ABD be the southern Half the *Sun's* Disk, C his Center, ND a Portion of the Ecliptic, E the Place of the Node in 1639, EH the Path of *Venus* at that Time, and CG her nearest Distance from the *Sun's* Center; then it is plain, if the Node E be at Rest, or continues still in the same Place, then also EH will now be her Path, the same as before; for the Angle CEO , which her visible Path makes with the Ecliptic, will always be the same, whether the Nodes move or not, as we shall shew hereafter.

It will also be shewn, that the Angle CEO is $8^\circ : 28'$, and since $CO = 4'$; therefore in the right-angled Triangle CEO we have this Analogy, as $\text{Sine } CEO = 8^\circ 28' : CO = 4 :: \text{Radius} : CE = 27'$ the Node from the *Sun's* Center. Then if we take $OK = OI = 1'$, we shall have $CK = 5'$, and $CI = 3'$; and then it will be $CO : CK :: (4 : 5 ::) CE : CN :: 27 : 33,8$; whence $CN - CE = EN = 6,8$ the Space through which the Node must apparently go forward, to cause *Venus* to pass one Minute more southerly in the Path NKG , or less so in the Path PIF , for $NE = EP$, as all her Paths are parallel.

In

XV. But for the Sake of such as are not thoroughly acquainted with the Doctrine of Parallaxes, the Doctor further explains the Matter, both by a Figure, and a somewhat more accurate Calculation: Therefore, supposing that at *London, May 25. 17^h 55'* 1761, the *Sun* be in $15^{\circ} 37'$ of *Gemini*, and consequently, that at his Center, the *Ecliptic* tends towards the North in an Angle of $6^{\circ} 10'$; and that the visible Path of *Venus* within the *Sun's* Disk does at that Time descend towards the South, forming an Angle with the *Ecliptic* of $8^{\circ} 28'$; the Path of *Venus* will tend a little towards the South in respect of the Equator, intersecting the Parallels of Declination in an Angle of $2^{\circ} 18'$: Supposing likewise, that *Venus* be near the *Sun's* Center at the said Time, and distant therefrom to-

The Doctor in his Tables places the ascending Node in $\Pi : 13^{\circ} : 37' : 44''$, at the Beginning of the Year 1661; and in the Year 1761 the Place of the same Node is $\Pi : 14^{\circ} : 29' : 24''$; the Motion therefore of the Node in 100 *Julian* Years will be $51' 40''$. From the Year 1639 to 1761 are 122 Years; therefore, as $100 : 51' 40'' :: 122 : 63' =$ Space thro' which the Node moves forwards in 122 Years; but the Equinox in the same Time recedes $99'$, therefore the ^{true} ~~apparent~~ Motion of the Node will be retrograde at the Rate of $36'$ in 122 Years: since therefore *Venus's* Distance from the Node in our Calculation is $26' : 36''$ (see Note * to Art. IX.) to which a geocentric Latitude of $4'$ corresponds; if we add the above Motion of $36'$ to $26' \frac{1}{2}$, the Sum is $62' \frac{1}{2}$, by which the Node will be distant from the Planet; and therefore since $26' 36'' : 4' :: 62 : 9 \frac{1}{2}$ nearly; therefore if the Doctor's Tables are right, we expect the nearest Approach of *Venus* to the *Sun's* Center to be about $9 \frac{1}{2}$ Minutes.

Here we cannot but observe how little was known, or said, of the Motion of *Venus's* Nodes about 40 years ago: It is certain the Doctor knew of no such Motion when he wrote this Essay; and I never could find the least Mention of any such Thing in any Pieces he afterwards published. Sir *Isaac Newton* positively denies such a Motion, and his learned Commentators the *Jesuits*, and other Astronomers since, as Mr. *De la Caille*, &c. Indeed a small Motion of the *Aphelia* and Nodes will accidentally arise from the perturbing Forces of the larger Planets; but this is too small to be regarded in the Theory of *Mercury* itself.

Nay, Dr. *Halley* has himself attempted to prove there is no sensible Motion of the Nodes of the Planets; for he found that of *Mercury* to amount to no more than $39' : 50''$ in 46 Years; and $38' : 20''$ of this was owing to the Recession of the Equinox, and therefore the proper Motion of the Nodes was only $1' : 30''$ in so long a Time. Therefore, says the Doctor, we may safely assume the Plane of the Orb of *Mercury* to be immoveable in the Sphere of the fixed Stars.

And indeed the Motion of the Nodes of *Mercury* which is found in his Tables, is only $1^{\circ} : 23' : 20''$ in 100 Years, which is the very same with that of the Equinox itself. The same Motion of the Node is also allowed to *Jupiter*; as also to the Node of *Mars*, though misprinted $1^{\circ} : 3' : 20''$. But for what Reason we have the Motion of *Venus's* Node $51' : 40''$, and the Motion of *Saturn's* Node $30'$, the Editors of those Tables have not thought fit to acquaint us. I fear there is too much Reason for their Exclamation — *Utinam Auctor, dum viveret, illas edidisset*. See further concerning the Motion of the Nodes, in a particular Discourse hereafter.

wards the South four Minutes, describing by a retrograde Motion on the *Sun's* Disk four Minutes an Hour; the *Sun's* Semi-diameter will nearly be $15' 51''$, and that of *Venus* $37''$ and $\frac{1}{2}$; and supposing, for Trial Sake, the Difference of the horizontal Parallaxes of *Venus* and the *Sun* to be $31''$, such as it is on the Supposition of the *Sun's* Parallax being $12''$ and $\frac{1}{2}$ *.

XVI. Therefore let a small Circle, as A B D Fig. 3. be described from the Center C, whose Semi-diameter let be $31''$, representing the *Earth's* Disk, and therein drawing D a b E and c d e the Ellipses of the Parallels of 22 and 56 Degrees N. Lat. in the same Manner as is now used by Astronomers for constructing Solar Eclipses; let B C A be the Meridian wherein the *Sun* is, to which let be inclined the Right Line F H, representing the Path of *Venus* in an Angle

* We are now come to the most material and essential Part of the Doctor's Essay, wherein the Transit is more directly explained by *Calculation, Projection, and Instrumental Mensuration*; but with so much Brevity, as to require some Degree of Illustration for duly understanding every Part.

The *Sun's* Place being given in *Gemini* $15^{\circ} : 37'$, there is given in a right-angled spherical Triangle, the Hypothenuse $= 75^{\circ} : 37'$, and the Angle at the Base $= 23^{\circ} 30' =$ Obliquity of the Ecliptic, to find the Angle at the Perpendicular, which will come out $83^{\circ} 50'$, the Complement of which is $6^{\circ} : 10'$, which Angle the Ecliptic makes with the Parallel of the *Sun's* Declination; and therefore is the *Direction of the Ecliptic Northwards* in that Point which the *Sun* then possesses.

It is farther said, that the visible Path of *Venus* makes an Angle with the Ecliptic of $8^{\circ} 28'$. To illustrate this; let N C be the Ecliptic, N the Node, and N B the proper Orbit of *Venus*, whose Inclination is the Angle C N B $= 3^{\circ} : 24'$. Then suppose the *Earth* and *Venus* were viewed from the *Sun*, and were at any Moment to set out together from the Node N, then in one Hour the *Earth* would appear to pass over the Space of $2' : 28'' = 148'' =$ N E; and in the same Time *Venus* will pass over $4' = 240'' =$ N V. Through the Point V draw the Line E A, and that will be a Direction or apparent Path of *Venus* to an Eye on the *Earth* in the Ecliptic at E. But in the Triangle N E V, there are given the two Sides N E and N V, and the included Angle at N, to find the Angle at V, which therefore is known (by Instit. 719). Then the Angle V E C $=$ N + V (by 632) $= 8^{\circ} 28'$. And because the mean Motions of the *Earth* and *Venus* are constant, the Spaces described by them in the same Time, will be always in the same constant Ratio, and therefore the Angle A E C will be invariable.

Since therefore the Ecliptic tends Northwards of the Equator $6^{\circ} : 10'$, and the Orbit of *Venus* tends Southwards of the Ecliptic $8^{\circ} : 28'$; therefore *Venus* will tend Southwards in Regard to the Equator in the small Angle of $2^{\circ} : 18'$. As to the Motions, Semi-diameters, and horizontal Parallaxes of the *Sun*, *Venus*, and the *Earth*, they are taken from the Tables for any given Time.

Note, If the Disk of the *Earth*, and the Position of *Venus's* Orbit were projected on a Plane, as seen from the *Sun*, then the Axis of the Ecliptic and that of *Venus's* Orbit would both lie on the same Side of the Axis of the Meridian B C A; in this Case, there is no retrograde Motion of the Planet; but since this retrograde Motion is

Angle of $2^{\circ} 18'$, whose Distance from the Center C let be 240 Parts, whereof B C is 31, and from C let fall the Right Line C H perpendicular upon F G; and supposing the Planet in H at $17^h 55'$ or $5^h 55'$ in the Morning, let the Right Line F G H be divided into the Hourly Spaces III. IV, IV. V, V. VI, &c. equal to C H, that is, four Minutes; let the Right Line K L be also equal to the Difference of the apparent Semi-diameters of the *Sun* and *Venus*, or $15' 13''$ and $\frac{1}{2}$; and the Circle described with the Radius K L, and from any Point within the small Circle, representing the *Earth's* Disk as a Center, will meet the Right Line F G in the Point denoting what o'Clock it is at *London*, when *Venus* shall touch the *Sun's* Limb in an Angle of internal Contact in that Place of the *Earth's* Superficies that lies under the assumed Point on the Disk; and if a Circle described from the Center C, and with the Radius K L, meet F G in the Points F and G, the Right Lines F H, H G will be $= 14' 41''$, which *Venus* will appear to pass over in $3^h 40'$; therefore F will fall upon $2^h 15'$ at *London*, and G upon $9^h 35'$ in the Morning; whence it is evident, that if the *Earth's* Magnitude should, by Reason of the immense Distance, vanish as it were into a Point; or if, divested of its diurnal Motion, it should always have the *Sun* vertical to the same Point C, the entire *Mora* of this Eclipse would continue for seven Hours and $\frac{3}{4}$; but in the mean Time, whilst the *Earth* revolves with a contrary Motion to that of *Venus* through 110 Degrees of Long. and consequently the Duration of the said *Mora* is shorter, suppose 12 Minutes, it will nearly be $7^h 8'$, or 107 Degrees*.

XVII.

is the Thing to be shewn, it is very easy to see that the Orbit of *Venus* must have a contrary Situation, and consequently that the Axis must be on the contrary Side of the Meridian, and therefore the *Difference*, and not the *Sum*, of the two Angles is to be taken for the Angle e C H, as the Doctor has prescribed.

* The PROJECTION the Doctor here speaks of, for constructing *Solar Eclipses*, is the *Orthographic* One (whose Principles are explained in the INSTITUTIONS of the GENERAL MAGAZINE, as being a Part of the SCIENCE of PERSPECTIVE). The Eye in this Projection is supposed to be placed in the *Sun's* Center, viewing the Motions of the Planets in their Orbits on a *Perspective Plane*; and therefore the Diameters of the Planets must be drawn in such Proportion as they appear on that Plane.

Let S be the *Sun's* Center (Fig. 2.) C that of the *Earth*, and M the Center of *Venus*, and suppose the Perspective Plane to pass through the Point M perpendicular to the visual Ray S M. Let A E D F be the *Earth*, and A C its Semi-diameter, subtending to the Eye at S, the Angle A S C $= 12'' \frac{1}{2}$ = *Sun's* horizontal Parallax. But it is evident the apparent Magnitude of A C on the perspective Plane at M will be H M. Parallel to H A (or S A) draw M B, then will the Angle B M C = A S C, and A B = H M.

D 2

Lastly,

XVII. Now in the Meridian itself *Venus* will be near the *Sun's* Center at the Eastern Mouth of the *Ganges*, where the Elevation of the Pole is about 22° ; therefore that Place will be equally distant from

Lastly, draw AM ; then is $AMC = \text{Venus's horizontal Parallax}$; from which if you take the *Sun's* BMC , there will remain the *horizontal Parallax of Venus from the Sun* $AMB = HAM$. Now in such small Angles we have $AC : AB :: AMC : AMB$; and consequently the *Earth's* Semi-diameter AC is diminished on the perspective Plane in HM in the Ratio of *Venus's* horizontal Parallax $AMC = 43'' \frac{1}{2}$ to her Parallax from the *Sun* $= 43'' \frac{1}{2} - 12'' \frac{1}{2} = 31''$, which therefore must be the Semi-diameter of the small Circle representing the *Earth's* Disk on the Plane at the Orbit of *Venus*.

But to an Eye at the *Sun*, *Venus's* Semi-diameter GM subtends an Angle $GS M = 15''$; and since $HS M = 12'' \frac{1}{2}$; therefore say, as $12'' \frac{1}{2} : 31'' :: 15'' : 36'' \frac{1}{2} = GM$ the Semi-diameter of *Venus* as measured in Seconds of a Degree on the same Plane; but the Doctor makes it $1'$ more, viz. $37'' \frac{1}{2}$.

Therefore, if from the *Sun's* Semi-diameter $15' : 51''$ we take that of *Venus* $37'' \frac{1}{2}$ there will remain $15' : 13'' \frac{1}{2}$ for the Difference. Then from a *Scale of equal Parts* take $951 = (15' : 51'' =) CL$, and on the Point C , as a Center, describe a Circle representing the *Sun's* Disk on the perspective Plane, Part of which is here denoted by SLU (Fig. 3.) then take in the Compasses $31 = AC$, and describe on the same Point C the small Circle $AEBD$ for the Disk of the *Earth*. Again, take $913 \frac{1}{2} (= 15' : 13'' \frac{1}{2}) = CI$, and describe the Arch VIY , whose Distance from the *Sun's* Limb will be $IL = 37 \frac{1}{2}$ the Semi-diameter of *Venus*.

Let CL be the Ecliptic on the *Sun's* Disk; then because it makes an Angle with the Meridian (passing through the *Sun* in the middle Moment of the Transit) of $83^\circ : 50'$; therefore make the Angle $LCH = 83^\circ : 50'$, and HCB , or rather AB , will be that Meridian on the Disk of the *Earth*.

Again, since the visible Path of *Venus* makes an Angle with the Ecliptic of $8^\circ : 28'$ whose Complement is $81^\circ : 32'$, therefore make the Angle $Lce = 81^\circ : 32'$ and Ce will be perpendicular to the Path of *Venus*, on the *Sun's* Disk. But since her Latitude is $4' = 240''$, therefore from the Scale take 240 , and set from C to e ; and through the Point e draw a Line Ge at Right Angles to Ce , and that will be the *visible Path of Venus on the Solar Disk*.

The Angle $eCH = 2^\circ : 18'$ being so small, the Doctor has neglected it in his Diagram, and supposed HC perpendicular to the Path of *Venus*; and since (by Calculation) the Middle of the Transit is at $5^h : 55'$ in the Morning; and also, at that Time, the horary Motion of *Venus* is just $4' = HC$; therefore, if you take the Line CH , and apply it Parallel-wise from 60 to 60 in the Line of Lines on the Sector; and then take the Distance between 55 and 55 on the same Lines, and set one Foot of the Compasses in H , the other will fall on the Point V , in the Line GH , in which Point the Planet will be at 5 o'Clock for the Time at *London*. Having this Point given, take in the Compasses the Line CH , and Place one Foot in V , the other will mark IV on one Side, and VI on the other; and in this Manner the other Hours III and II are found; and thus the Line GH is divided into Hours, and then each Hour may be sub-divided into Minutes, as you see in the Figure.

The Line GH will cut the Curve VIY in F , whereon, if a Circle be described with the Radius $= IL = 37'' \frac{1}{2}$, it will represent the Disk of *Venus* just entered the *Sun*, and

from the *Sun* on both Hands in the Moments of the Planet's Ingress and Egrefs, viz. 53° and $\frac{1}{2}$, as the Points $a b$ in the greater Parallel $D a b E$; but the Diameter $A B$ will be to the Distance $a b$, as the Square of the Radius to the Rectangle under the Sines of 53° and $\frac{1}{2}$ and 68° ; that is, as $1' 02''$ is to $46'' 13'''$; and upon making a due Calculation, the Doctor finds, that the Circle described with the Radius $K L$ from the Center a , will meet the Right Line $F H$ in the Point M , at $2^h 20' 40''$; but described from the Center b , it will meet $H G$ in N at $IX^h 29' 22''$ at *London*; consequently the whole Body of *Venus* will be seen from the Banks of the *Ganges* within the *Sun's* Disk for $7^h 8' 42''$; therefore we have rightly supposed its Duration $7^h 8'$, since here a Part of a Minute is inconsiderable*.

XVIII.

and in internal Contact with its Limb, such as it appears in that Case from the Center of the *Earth* at C ; for the Line $C F = C I$. And because in the Right-angled Triangle $C F H$, there is given the Side $C H = 240''$, and $C F = 913'' \frac{1}{2}$; we shall have $F H = \sqrt{C F^2 - C H^2} = 880''$; then say, as $240'' : 1^h :: 880'' : 3^h 40'$; which deducted from $5^h : 55'$, will leave $2^h : 15'$ for the Time of the Morning when *Venus* will be just within the solar Disk; but added thereto, it makes $9^h : 35'$, the Time when the Egrefs begins, and consequently the whole Duration of the Transit to an Eye at the Center C , or at Rest on the Middle of the *Earth's* enlightened Disk, would be just 7 Hours and 20 Minutes, in which Time 110 Degrees of Longitude will pass the Meridian.

* But, as we have shewn, the Surface of the *Earth* being in Motion, and in some Parts that Motion conspires with, and in others it is contrary to that of *Venus*; it follows, that in the latter Case there will be some Places, as (a) and (b) where the Transit will be of a less Duration, and others as (c) and (d) where it will be longer than at the Center C .

Suppose by the contrary Motion from (a) to (b) in the Parallel of $22^\circ : 30'$, the *Mora* be contracted 12 Minutes, or that it lasteth but $7^h : 10'$, then will the Arch $a b = 107$ Degrees, and consequently $a C = 53^\circ : 30'$ in that Parallel. Now in order to determine the Time of the Beginning at the Point (a) we must illustrate the Principles of Calculation and Projection by a larger Figure of the *Earth's* Disk.

Therefore from a larger Scale of equal Parts, take in your Compasses 31, and on the Point C describe the Semicircle $F B G$ (Fig. 4.) and project the Elliptic Meridian and Parallel for the given Point (a) , whose Position on the Disk is then determined by Calculation in the following Manner. Since the Arch $D G = 22^\circ : 30'$, its Complement $D B = 67^\circ : 30'$. Then we have Radius : Sine of $D G :: C G :: D O :: 31 : 11,8 = D O = C H$. Again, $R : s D B :: 31 : 28,74 = H D$. Then because $C G : H D :: R : s D B$; and $H D : M a :: R : s C a = 53^\circ : 30'$; therefore by Composition of Ratios, we have $C G : M a :: R^2 : s B D \times s C a$; whence $M a = 23,1$.

The Arch $a D = 36^\circ : 30'$, therefore $R : s D a :: H C : K a = H M :: 11,8 : 7,1$. Whence $C H - H M = 4,7 = M C = a N$. Having therefore $M a$, and $a N$, the Point (a) is given in Position on the Disk.

Therefore (in Fig. 3.) from (a) let fall the Perpendicular $a g$, then will it be $240 + 4,7 = 244,7 = a g$. Also $H g = M a = 23,1$; then say, as $240 : 60' :: 23,1 : 5'$:

XVIII. But adapting the Calculation to *Nelson's Harbour*, the Doctor finds that *Venus* shall pass under the *Sun's* Disk when he is just about to set, and emerge out of his Disk immediately after his rising; that Place, in the mean Time, being carried through the Hemisphere opposite to the *Sun* from *c* to *d*, with a Motion conspiring with that of *Venus*; therefore the *Mora* of *Venus* within the *Sun's* Disk will become longer by reason of the Parallax, suppose by four Minutes, so as intirely to be $7^h 24'$ or 111° of the Equator; and since the Latitude of the Place is 56 Degrees, it will be as the Square of the Radius to the Rectangle under the Sines of 55° and $\frac{1}{4}$ and 34° , so is $AB = 1' 2''$ to $cd = 28'' 33'''$, and upon duly making the Calculation, it will appear, that the Circle described from the Center *c*, with the Radius *K L*, will meet the Right Line *F H* in *O* at $2^h 12' 45''$; but described from the Center *d*, it will meet *H G* in *P* at $IX^h 36' 37''$; wherefore the Duration of the *Mora* at *Nelson's Harbour* will be $7^h 23' 52''$; to wit, greater than at the Mouth of the *Ganges* by $15' 10''$ of Time*.

XIX.

$5' : 48''$, which added to $5^h : 55'$ gives $6^h : 0' : 48''$ for the Time when the Planet will be at *g*.

The Point (*a*) being determined on the Disk; take the Distance *C I* in your Compasses, and setting one Foot in (*a*) describe the Arch *W Z*, which will cut the Path of *Venus* in *M*, and thereby shew the Time of the entire Ingress as seen from the Place under (*a*) to be at $2^h : 20' : 48''$. This also appears from Trigonometrical Calculation; for the Triangle *a M g* is nearly the same as *C F H*; and therefore, $g M = H F$; whence $H g = F M = 5' : 48''$, which added to the Time at *F* $= 2^h : 15'$ gives $2^h : 20' : 48''$, the Time at *M*. Hence twice $5' : 48''$, or $11' : 36''$ will be the Time by which the Transit will be shortened at the Parallel of $22^\circ : 30'$.

* Again (in Fig. 4.) let (*c*) be in the Parallel of 56° , and the Motion of the Earth there conspiring with that of *Venus*, the Transit will be thereby prolonged, which suppose to be 4 Minutes. Therefore to the whole Duration $7^h 24'$ there will answer 111° of the Equator, or in the Arch of the Parallel between (*c*) and (*d*), the Half of which is $55^\circ : 30'$, also the Co-Latitude is 34° . Then proceeding as above, we have $R : s 34^\circ :: C F : g e :: 31 : 17,3 = g e$. And as $R : s 56^\circ :: C F : C g :: 31 : 25,7 = C g$. Also, $R : s 55^\circ 30' :: e g : c m :: 17,3 : 14,3 = c m$. And because of the Circles *E C D* and *e p f* being parallel, we have $H D : g e :: H C : g p = 7,1$. And then, lastly, we have $R : s 34^\circ 30' = c e :: g p : c n = g m :: 7,1 : 4 = g m$; whence $C g + g m = 29,7 = C m$.

If therefore from the Point (*c*) (Fig. 3.) we let fall the Perpendicular (*c f*) then $f H = (c m) = 14,3$, and then $240 : 60' :: 14,3 : 3' 33''$, which taken from $5^h 55'$, leaves $5^h 51' 27''$, the Time of the Morning when *Venus* will be at (*f*). Then taking the Line *C I* in the Compasses, and setting one Foot in (*c*) describe the Circle *T X*, this will cut *Venus's* Path in *O* at $2^h 13' 27''$; the Time when *Venus* will appear at (*c*) just within the solar Disk.

Or

XIX. But if *Venus* should pass without Latitude, the said Difference will become $18' 40''$; but if she shall be four Minutes more northerly than the *Sun's* Center, the Difference will be increased to $21' 40''$, and will be still greater by increasing the Planets N. Lat.

From the above Hypothesis it follows, that at *London Venus* shall rise entering into the *Sun*, and at $9^h 37'$ in the Morning, in her Egrefs, touch internally the *Sun's* Limb, and quite leave his Disk not before $9^h 56' \dagger$.

XX. It is evident from the same Hypothesis, that *Venus* should touch with her Center the extreme Northern Limb of the *Sun* on *May 23*, $11^h 1769$; so that, by Reason of the Parallax, her whole Body may be seen, in the Northern Parts of *Norway*, within the *Sun's* Disk; whilst on the Coast of *Peru* and *Chili*, she shall seem to ride on the Disk of the setting *Sun*, with a small Segment of

Or thus by Calculation; in the Right angled Triangle $c O f$ there is given the Perpendicular $c f = C H + C m$ (Fig. 4.) = 269,7, and the Hypothenuse $c O = 913,5$; therefore the Base $O f = \sqrt{O c^2 - c f^2} = 872,5$. Then say, as $240 : 60' :: 872,5 : 2180' = 3^h 38'$; which taken from $5^h 51' 27''$ leaves $2^h 13' 27''$, the Time at (*c*) as before.

Wherefore the Time of the intire Ingress at (*c*) *Port Nelson* will precede the same at (*a*) *Bengal* by about $7' 21''$; and therefore the whole Duration will be near $15'$ longer in the former Place than in the latter.

\dagger But if the Node should be in the *Sun's* Center *C* (or the Transit be central), and the Path of *Venus* be *N C*; then it is evident that $f g = P R$ (Fig. 3.) = $M a + c m$ (Fig. 4.) = $23,1 + 14,3 = 37,4$; because in this Case, the Distance of (*a*) from the Line *N C* nearly vanishes, and that of (*c*) is inconsiderable. Therefore say, as $240 : 60' :: 37,4 : 9',35 = 9' 21''$; the Difference of Time in the Beginning of the Transit at *Bengal*, and *Nelson's Harbour*, the Double of which, viz. $18' 42''$ will be nearly the Difference of the whole Durations in this Case.

If *Venus* should pass with 4 Minutes of North Latitude, then, by adapting the Trigonometrical Calculus, as before for 4' South Latitude, the Difference of the *Mora* in those Places will be found $21' 40''$. And, indeed, as she goes more Northerly, the Difference will encrease as you may plainly see in the Diagram, the Distance *P R* is greater than *O M*, and *T W* is greater than *P R*, because the Arch *T X* does sensibly recede from the Arch *V Y* as it goes Northwards; making the Difference *O F*, *P Q*, *T V* to encrease to a certain Limit, while the other Parts *F M*, *Q R*, *V W*, vary but little on all this Part of the Disk.

Since on that Day at *London* the *Sun* rises but a few Minutes before *IV*, therefore she will be advanced upon the *Sun's* Disk at his Rising to the Place in her Path where you see her represented in the Scheme. So that near two Hours of the Transit will be lost to us.

As the Diameter of *Venus* is $75''$, if we say as $240 : 60' :: 75 : 18'$, $75 = 18' 45''$, that will be the Time which passes between the external and internal Contact with the *Sun*, or so long as she will be transiting the *Sun's* Limb.

her

her Body; as in like manner in the *Molucca* Islands, and neighbouring Parts, at Sun-rising: But if the Nodes of *Venus* be found to have a Retrocession (as there is Reason to suspect from some later Observations) then her whole Body being every-where seen within the *Sun's* Disk, the greatest Difference of these Eclipses will afford a still more evident Proof of the *Sun's* Parallax †.

XXI. How, from Observations made in the *East Indies* on the Ingress and Egress of *Venus*, compared with those which are made in *England* on her Exit from the Solar Disk, the same Parallax may be derived; to wit, *by adapting the Angles of a Triangle given in*

* This Paragraph relates to the Transit of *Venus* in the Year 1769, on *June* 3d, at eleven o'Clock at Night, and therefore invisible to us. Upon the Hypothesis that the Nodes are at rest, the Center of *Venus* will nearly touch the Northern Part of the *Sun's* Limb, because the Latitude of *Venus* will in that Case be $15' : 43''$, and the *Sun's* Semi-diameter is then about $15' 50''$. (See the Table in Note II, Art. IX.)

But, by an apparent retrograde Motion of the Nodes, the Planet will pass with less Latitude; and if that Motion be such as results from some modern Observations, the Transit will be very considerable, and of much greater Use in determining the Parallax of the *Sun* than that of next *June*.

CASSINI, in his Elements of Astronomy, has collected the following Observations of the Motions of the Venereal Nodes.

From an Observation of *Timocharis*, 271 Years before Christ, of *Venus* eclipsing the Star η in the Southern Wing of *Virgo*, it is computed that the Node of *Venus* was at that time in $8 : 24^{\circ} : 2'$.

By the Observation of *Horrox*, in 1639, the Place of the Node was found to be in II $13^{\circ} : 28' 22''$.

By an Observation made in 1698, it was found by Calculation, that the Place of the Node was found in II $14^{\circ} : 1' : 45''$.

By another Observation in the Year 1705, the Place of the Node was found in II $14^{\circ} : 2' : 52''$.

In the Year 1710, the Node was found in II $14^{\circ} : 04' : 52''$.

And in the Year 1731, the Place of the Node was in II $14^{\circ} : 17' : 02''$.

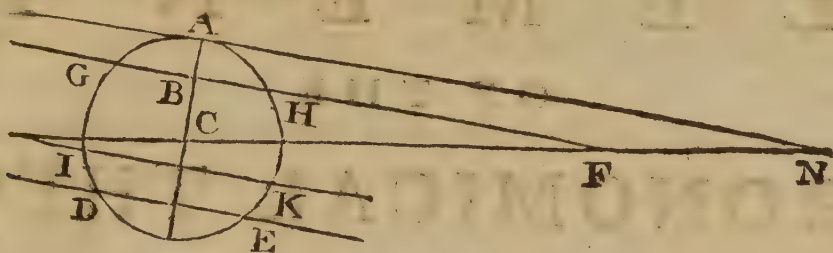
From these Observations you will easily find that the Motion of the Node is at a Mean, $34''$ per Annum. But Dr. Halley's Tables make it $31''$ yearly.

Also it is evident, that $1731 + 271 = 2002$, or from the Time of *Timocharis*, to the Year 1731, there are 2002 Years, in which Time the retrograde Motion of the Equinox, or Node of the *Earth*, is $27^{\circ} : 48' : 20''$; it is plain, if the Nodes of *Venus* were fixed, they would have appeared to have moved so much forwarder in the Ecliptic; but they have appeared to go forwards only $20^{\circ} : 15' : 2''$; therefore the Difference $7^{\circ} : 33' : 18''$ is the true retrograde Motion of the ascending Node in that Time.

Lastly, it must be observed, that since the Orbit of *Venus* lies within that of the *Earth*, the real retrograde Motion of the ascending Node will produce an apparent direct Motion of the descending Node, which is the Reason why the Motion of this Node is contrary

in Specie in the Circumferences of three equal Circles, I shall take another Opportunity to shew ‡.

contrary to that of *Venus* in all the Diagrams we have hitherto made use of, as also in that which follows, for a further Illustration of this most important Affair.



Let A D E be the *Sun*, C N the *Ecliptic*, A N the *Path of Venus*, and N the *Node*. Now if the *Node* be at *Rest*, the nearest *Distance* of the *Planet* from the *Center*, will be $A C = 15' 43''$. And since the *Angle* $A N C = 8^{\circ} 28'$, we shall find $C N = 106$ the *Distance* of the *Node* from the *Center* C.

But if the Nodes have a retrograde Motion, and equal to that in the Doctor's Tables, then since from the last Transit in 1639, to that in 1769, are 130 Years, this Node will appear to have gone forward in that Time $38^{\circ} 30'$ nearly; therefore $106 - 38,5 = 67,5 = \text{F C}$, and F will be the Place of the Node in this Hypothesis.

Then drawing F G parallel to A N, it will be the Path of the Planet; and B C its Distance from the Sun's Center, which is known from this Analogy, as $106 : 15,75 :: 67,5 : 10 = B C$. Therefore the Path of *Venus* G H, upon the solar Disk in this Transfit, will be very little less than D E in the next Transfit 1761, according to the Doctor's Tables.

But the Transit G H will be greatly preferable to the Transit D E for determining the *Sun's* horizontal Parallax, because, as we have shewn, a Transit on the North Part of the solar Disk, will give the Difference of Duration in the *Mora* greater than one in the southern Part, every Thing else being equal; and the Transit G H happens to be nearly in that Part of the Disk where that Difference of Time will be a *Maximum*, and consequently will afford the best Opportunity that can be for Success in this Discovery.

‡ Whether the Doctor did ever give the Solution of this Problem we know not, nor have we met with it in any other Author. We shall therefore leave it to our learned Correspondents (in our MAGAZINE) to oblige the Public with it.

T H E
E L E M E N T S
O F T H E
A S T R O N O M I C A L T H E O R Y
O F T H E
P L A N E T V E N U S,
W I T H R E G A R D T O
C A L C U L A T I O N S f r o m T A B L E S.

IT is impossible for any one to understand the Nature of the Tranfit of *Venus*, and the Computations and Calculations relative thereto, without being previously acquainted with the Astronomical Principles of this Planet's Motions, which, therefore, I shall here briefly explain. Let $\gamma \subseteq \approx \wp$ represent the ECLIPTIC or Path of the *Earth* about the *Sun* S (see Figure 1. Plate III.), in which let the *Earth* be at T, and suppose the Distance T S divided into 100,000 equal Parts; then by the known Laws of Motion, the Squares of the periodical Times in any two Planets are always proportioned to the Cubes of their mean Distances; and since the Times in which the *Earth* and *Venus* revolve about the *Sun* are known, and the *Earth's* Distance T S is equal to 100,000, therefore the Distance of *Venus* will be found, by the above Analogy, 72,333 of the same Parts, and consequently if from a Scale of equal Parts we take that Number, and on the Center S describe the dotted Circle B \oslash C \wp . Then that would be a proper Representation of the Orbit of *Venus*, if that Planet moved in the Plane of the Ecliptic, and in a circular Orbit.

But

But as her Orbit is inclined to the Plane of the Ecliptic in an Angle of $3^{\circ} 24'$ nearly, one Part thereof will lie above the Plane of the Ecliptic, as α E G ϑ ; and the other below, as α H ϑ ; and the common Interfection, or Line ϑ S α , is called the Line of Nodes, and the Point α is called the *ascending Node*, and the other Point ϑ the *descending*, because at the former she ascends above the Plane of the Ecliptic, and in the latter she descends below it. The Place of the ascending Node α is in $14^{\circ} 30'$ of *Gemini* in the present Age, and as here represented in the Figure.

As the Orbit of the Planet is situated within that of the *Earth*, she will be seen twice in Conjunction with the *Sun*, viz. in the Points R, Q, which are in a right Line drawn from the *Earth* to the *Sun*; that at R is called the *superior Conjunction*, and that at Q the *inferior*.

Supposing her Orbit circular, as Q C R B, and in the same Plane of the Ecliptic; then if two right Lines were drawn from the *Earth* at T, to touch the Orbit of *Venus* in the Points I, K, they would determine the Angle of greatest Distance that this Planet could ever appear from the *Sun*. Thus as she descends from her superior Conjunction R, she will appear to go further from the *Sun* continually, till she arrives to the Point I, and there she will appear at the greatest Distance from the *Sun* at S; afterwards, as she advances towards her inferior Conjunction Q, the Distance from the *Sun* gradually decreases. From Q her apparent Distance from the *Sun* again increases till she arrives at K, and from thence decreases as she advances by C towards R.

This Angle I T S of her greatest apparent Distance, is called the Angle of her *greatest Elongation* from the *Sun*: And because the Triangle S I T is right-angled at I, and the two Sides S T and S I are known, from thence we find the Angle S T I of about 48° which therefore is the greatest Distance she can ever be elongated from the *Sun*.

It is farther evident, that to a Spectator at the *Earth*, the Planet *Venus*, all the while she is moving through the Part of her Orbit K C R B I, will appear to be *direct* in Motion, or to move according to the Order of the Signs from West to East. But at the Point I the Direction of her Motion is changed; and during her Passage from I, by Q to K, her Motion will be contrary to the Order of the Signs, or from East to West; and therefore she is, during this Time, said to be *retrograde*.

Hence also it appears, that since the large Part of her Orbit $K R I$, and the smaller Part $I Q K$, are both seen or described under the same Angle $I T K$, therefore her Motion in the former Case must be very slow in Comparison of the latter; and as at the Points $I K$, her Direction is for a little Time before and after, sensibly in the right Lines $I T$, $T K$, and consequently she must be seen in the same Part of the Heavens during those times, therefore she is then said to be *stationary*.

Were the Earth at rest in T , the Space of Time between two Conjunctions of the same Kind, would be equal to the periodical Time of the Planet, which is nearly 225 Days; but since both the *Earth* and *Venus* move together the same Way, this Time between the two Conjunctions will be very much enlarged beyond the periodical Time of either: For if the Conjunction be at Q , then the Planet must perform one Revolution, and so much of another as is equal to the Motion of the *Earth* between the Times of the two Conjunctions. Let the *Earth's* Motion in that Time be called A , then it will be * as $8766^h : 5392' :: 360^\circ + A : A$; and by Division of Proportion $8766 - 5392 (= 3374) : 5392 :: 360 : A$. And therefore $A = \frac{5392 \times 360}{3374} = 575$ Degrees nearly; which angular Motion the *Earth* makes in one Year, seven Months, three Weeks and one Day, which is the Time sought between the two Conjunctions.

The Particulars we have now mentioned are upon the Supposition that the Orbit of *Venus* is in the Plane of the Ecliptic; but it makes very little Difference if we consider her Orbit as inclined thereto in the small Angle above-mentioned.

We have likewise hitherto spoken of her Orbit as Circular, but it is in reality Elliptical; in one of whose Focus's the *Sun* is placed.

The longer Axis of her Ellipsis cutting the Ecliptic in about the 7th Degree of *Leo* and *Aquarius*, as in the Figure is shewn by the Line $M V$, and upon her Plane $A S$ is her greatest, and $E S$ her least Distance from the *Sun*; and therefore the Point A is called the *Aphelion*, and E the *Perihelion*; and the greatest Distance $S A$ is to the least Distance $S E$, as 72857 is to 71823.

* The Period of the *Earth* is $365^\circ : 6^h = 8766^h$, and the Period of *Venus* $224^\circ : 16^h = 5392^h$.

N. B. The Theorem from whence the above Analogy is derived, you may see in Instit. 244 of the *General Magazine*.

Let P be the Place of a Planet in its Orbit, and let fall the Perpendicular P L to the Plane of the Ecliptic; then is L, the Place of the Planet, *reduced to the Ecliptic*. Draw S P and S L, and they will contain an Angle P S L, which is called the *heliocentric Latitude* of the Planet, because the real Latitude P L of the Planet from the Ecliptic, is thereby measured as seen *from the Sun*. Draw the Line T P and T L, and they will contain an Angle P T L, under which the Latitude of the Planet P L is seen *from the Earth*, and therefore this Angle is called the *geocentric Latitude*, and these two Angles are inversely as their Distances from the Planet; that is, the heliocentric Latitude P S L is to the geocentric Latitude P T L, as the Distance of the *Earth* T P is to the Distance of the *Sun* S P from the Planet.

The Distance of the Planet P from the Node, or the Arch P γ , is called the *Argument of Latitude*. Suppose T D the greatest Latitude; that will measure the Inclination of the Planet's Orbit to the Ecliptic, as it will differ not sensibly from the Arch of which it is the Sine; therefore we shall have the following Analogy; as Radius to the Sine of the Planet's Distance from the Node, so is the Inclination of the Orbit to the Latitude P L of the Planet at P; which Latitude seen from the *Earth*, will always be of a different Quantity, according to the different Distances of the *Earth* from the Planet.

The Distances of the Planet from the *Sun* and the *Earth* in the Ecliptic, *viz.* S L and T L, are called the *curtate Distances*; and in the Triangle T S L, in the Plane of the Ecliptic, the Angle T S L is called the *heliocentric Longitude* of the Planet from the *Earth*; as being the Difference between the Place of the *Earth*, and that of the Planet (reduced to the Ecliptic), seen from the *Sun*. The Angle L T S is the *geocentric Longitude* (but more commonly called the *Elongation* of the Planet from the *Sun*), which is the Difference between the Place of the *Sun* and of the Planet in the Ecliptic, as seen from the *Earth*. Lastly, The Angle S L T is called the *Parallax of the annual Orbit*, as being subtended by the Semi-diameter S P of the *Earth's* Orbit. Note, The Angle T S L is commonly called the *Angle of Commutation*.

Not only the Orbit of the Planet *Venus*, but those of all the other Planets, are Elliptical, and particularly that of the *Earth*; and therefore the Distances of the *Earth* from the *Sun* will always be

be unequal; the greatest Distance will be 10169, and the least 9831, the mean Distance being 10,000, as we mentioned before.

The Longitude, or Place of the *Earth* or Planet, is its Distance in the Ecliptic from the Beginning or first Point of *Aries*; and what is called the mean Anomaly, is the Distance from the Aphelion Point, as from the Point A in the Orbit of *Venus*; and is estimated in Signs and Degrees of the Ecliptic.

From what has been said, it will be easy to understand the Process of Calculation by Astronomical Tables; in which we find the Longitude, the mean Anomaly, the Place of the Aphelion and Node, the Equation of the Orbit, the curtate Distances, the Argument of Latitude and Reduction to the Ecliptic, all ready calculated for the *Earth* or *Sun*, and all the other Planets; so that by this Means we are enabled, at any Time, to find the geocentric Place, Latitude, and Distance of the Planets, as Occasion shall require; an Example of which Method of Calculation you will see in Page 9. of Dr. *Halley's Dissertation*, and it is the same with regard to any other Tables, as well as the *Caroline* there made use of.

O F T H E
 N O D E S o f V E N U S,
 A N D T H E I R
 M O T I O N a n d P L A C E i n t h e E C L I P T I C .

AS the *Place* and *Motion* of the Nodes of *Venus's* Orbit are the two Hinges on which the whole Doctrin of the Transit turns, they cannot be too well ascertained and understood; and therefore we may a little wonder, considering what a Number of Observatories have been erected in the different Parts of *Europe* for a Century past, that our Observations on the Place and Latitude of *Venus* are so very few, and made by so small a Number of Persons. I, for my own Part, have seen little on this Head, besides what is contained in *Cassini's* Astronomy, either in regard to the Place or Motion of the Nodes of *Venus*.

It is evident from the foregoing Theory (Plate III. Figure 1), That if an Observation be made of a Planet's Place and Latitude, as seen from the *Sun*, and this be done on each Side of the Node Ω , before and after its Passage through it, that then the Place of the Node Ω may be easily determined by Calculation; and these Observations being repeated at distant Times, will give the Motion of the Node and its Direction. It may be proper to illustrate this by an Example, which I shall give from the before-mentioned Author.

On the 2d of *September*, at 15 Minutes after Two (N. S.) the true Place of *Venus* was observed in $\approx 16^{\circ} 30' 50''$ with North Latitude $5' 41''$. Afterwards, on the 6th Day of the same Month, her Place was observed at 18' after Two to be in $\approx 21^{\circ} 18' 20''$, with South Latitude $7' 13''$; then having computed for these Observations the true Places of *Venus* seen from the *Sun*, it will be found that her Place for the First was in $\approx 11^{\circ} 19' 2''$, and the Second \approx

$17^{\circ} 40' 23''$, and then the Place of the Node is found by the following Analogy; as the Sum of the Two Latitudes $12' 54''$ is to the Latitude of the first Observation $5' 41''$, so is the Motion of *Venus* between the two Observations (which was $6^{\circ} 21' 21''$) to $2^{\circ} 48'$; which being added to the Place of *Venus* in the first Observation, gives the Place of the Node in $\rightarrow 14^{\circ} 7'$, which is the descending Node, because her Latitude at the first Observation was North, and at the second South.

Again, on the 7th of *September*, at 19 Minutes past Two, her geocentric Place was found to be in $\rightarrow 22^{\circ} 27' 30''$ with $11' 31''$, and by Computation her heliocentric Place for the same Time was in $\rightarrow 19^{\circ} 15' 30''$; now comparing this with the first Observation on the 2d of the same Month, we have the following Analogy; as the Sum of the two Latitudes $17' 12''$ is to the first Latitude $5' 41''$, so is $7^{\circ} 56' 28''$ (the Motion of *Venus* in the Interval) to $2^{\circ} 37' 30''$. These being added to *Venus's* true Place in the first Observation, give the Place of her Node in $\rightarrow 13^{\circ} 56' 30''$, which is $10' 30''$ more than was found by the preceding Analogy; and therefore taking the Mean, we shall have the Place of the descending Node, on the 4th of *September* 1698, in $\rightarrow 14^{\circ} 1' 45''$.

Afterwards other Observations were made in the Year 1731, on the 7th and on the 14th Day of *April*, and from those Observations the Place of the same Node was determined, in the Manner above specified, to be in $\rightarrow 14^{\circ} 17' 2''$, more advanced by $15' 17''$ than on 4th of *September* 1698, which is at the rate of $28''$ per *Annum* for the Motion of *Venus's* Nodes.

By other Observations made by the Moderns, by *Horrox* at the last Transit, and by *Timocharis* 271 Years before the *Christian Æra*, our Author has made such Calculations as determine the annual Motion of *Venus's* Node to be at a Mean $34''$, per *Annum*.

When such Observations and Calculations were so easy, and already furnished to their Hands, one may very well wonder that some of our greatest Astronomers should be altogether silent about the Motion of the venereal Nodes. Indeed, in the Astronomical Tables of *PTOLEMY* and *KING ALPHONSUS*, there is no Mention made of the Nodes of *Venus*, or their Motion: But in Tables that were published more than 100 Years ago, viz. by *Jeremiah Shakerley*, in 1653, intituled, *Tabulæ Britannicæ*, we find the Place of *Venus's* Node is considered and allowed for, at the Rate of $50' 16''$ in 100 Years, which is at the Rate of $30''$ per *Annum*.

The

The RUDOLPHINE Tables, compiled from the Observations of Tycho, Kepler, and others, do also allow for the Motion of Venus's Node $78' 10''$ per Century, which is at the Rate of $47''$ per Annum. These were compiled and published by John Baptista Morinus, Regius Professor of Mathematics at Paris, in the Year 1651; concerning which the Author assures us, that the Places of the Planets, as found by his Tables, differ not a Minute from their Places found by the Tables and Methods deduced from Kepler's Observations.

In the Year 1661, those called the CAROLINE Tables were published by Mr. Thomas Street; but in them we find no Mention of Venus's Nodes, or their Motion, although he relates many Observations that were made of this Planet by Tycho, Mæstlinus, Kepler, Gassendus, and others.

In the Year 1702, Mr. De la Hire, Regius Professor of the Mathematics at Paris, published his *Astronomical Tables* at the Command of Lewis the Great, in which he allows a Place to the Node of Venus, and a Motion of $1^{\circ} 16' 48''$ per Century, which is $46''$ per Annum; but a very little differing from the Rudolphine Tables.

Soon after this another Collection of Tables was published by Mr. Whiston, said to be those of Mr. Flamsteed, Dr. Halley, Mr. Cassini, and Mr. Street; in this Collection there is no Mention made of the Nodes of Venus, or any Motion considered; nor is there for any other of the Planets. What those Tables were which he calls Mr. Flamsteed's or Mr. Cassini's, I know not: But as for those published under Dr. Halley's Name, and by Cassini the younger, they allow for the Nodes of Venus, and for the Nodes of the rest of the Planets.

In Dr. HALLEY's Tables, which were printed in the Year 1719, the Motion of Venus's Node is stated at $51' 40''$ in 100 Years, which is at the Rate of $31''$ per Annum.

Lastly, The Table published by Cassini the Son, in the Year 1740, contains, perhaps, the truest Account of the Motions of the Planet's Nodes in general, and that of Venus in particular; it makes that to be $57' 40''$ in 100 Years, which is at the Rate of $34''$ per Annum, and therefore, if the Transit of Venus be calculated by those Tables of Mr. Cassini, the Phases and Times thereof will be different from those which are calculated by Dr. Halley's Tables, and probaby nearer the Truth.

T H E
DOCTRINE OF PARALLAXES
M O R E F U L L Y
E X P L A I N E D.

AS we purpose to shew and explain the Method of determining the various Phænomena of the Transit by means of Parallaxes, as the most natural and truly astronomical of any, it will be previously necessary to give the Reader as distinct an Idea of this important Point as we possibly can.

We have already shewn, that by the Word Parallax nothing more is meant than the Difference of Place in which any Object is seen from two different Points of View. Thus if the Planet V (Fig. 2.) be viewed from the Center of the *Earth* at T, its apparent Place will be at C, in a Plane F G placed indefinitely beyond it in the Heavens. But if the same Planet be viewed from the Surface of the *Earth* at E, its apparent Place will be at D; and this Difference, C D, of apparent Places, is called the PARALLAX of that Object in the Heavens.

Let K L M represent the *Sun*, and let the Visual Rays T C, E D, in their Way to the Plane F G, cross a Diameter of the *Sun* in the Points H I; then will the Distance H I be the Difference of the apparent Places of the Planet on the Face of the *Sun*. If the Spectator's Eye be placed at the Point (e), the Planet will appear on the Plane F G at the Point (c), and the Difference of the apparent Places of the Planet seen from the Center of the *Earth* T, and by a Spectator at (e) will only be C c. Lastly, a Spectator at the *Earth*'s Surface at O, which is in the right Line T V, joining the *Earth* and Planet, must necessarily observe the Planet in the same Point C, as if he had viewed it from the Center of the *Earth*; and in this Case, as there can be no Difference of apparent Place, there can be no Parallax at all.

From what we have said it is evident, that the Difference of the apparent Places C D (or C c), or the Quantity of the absolute Parallax

rallax is proportional to the Angles TVE and TVe , which Angles are therefore called the *Parallatic Angles*, or Angles which measure the *Parallax*. These Angles are proportioned to the Sines of the Arch Oe and OE , or to the Distances of the Spectator from the Point O , to which the Planet is vertical. It is evident, when the Arch OE is 90° , or when the Planet appears in the Horizon, that then the Angle EVT , or the Parallax, is greatest of all; and is in that Case called the *Horizontal Parallax*, being that under which the Semi-diameter of the *Earth* is seen from the Planet, and of which we have more particularly treated in the Note to the VIIth Paragraph of Dr. HALLEY's Dissertation, Page 6.

The Parallax hitherto considered is properly called the *Absolute Parallax*, and has respect to one Point V , seen from two different Places. But there is another Sort or Distinction of a Parallax, which may be properly called the *Relative Parallax*, which has Respect to the different Places in which two Objects appear from one given Point T . Thus the Center V of a Planet, and the Center S of the *Sun*, seen from the Center of the *Earth* T , appear at C and A , and when viewed from the Point E , they appear at D and B . But since the Distance of the Planet TV , and that of the *Sun* TS , are very unequal, the Parallax AB of the *Sun*, and that of the Planet CD , will be unequal in Proportion, or the Center of the *Sun* S will have a much less Depression from A to B than that which the Center of the Planet has from C to D , therefore these two Centers of the *Sun* and Planet are relatively depressed, as viewed from the Point E thro' a Space which is equal to $AD - AB$; therefore make Db equal to AB , then will Ab be the Quantity of Depression, or *relative Parallax*, by which the Center of the Planet V is removed from the Center of *Sun* S upon the Plane FG , and this is called the *Parallax of the Planet from the Sun*; and when we speak of the apparent Distance of the Planet from the *Sun's* Center on the Face of the *Sun*, we consider the Distance SI to be diminished in the Ratio of AD to Ab on the Plane, or we take the Difference between the absolute Parallax of the Planet HI , and that of the *Sun*, whenever we make any Computation of the Distance of a Planet from the Ecliptic by means of the Parallax, as we shall hereafter more fully explain and illustrate, by an Application of this Doctrine to the Transit observed by *Horrox*, in the Year 1639, having first premised the general Theory of Calculation by Parallaxes, which next follows.

T H E
COMPUTATION by PARALLAXES

EXPLAINED; and APPLIED to the

TRANSIT of V E N U S,

For Ascertaining the TIMES of its Duration in different
Parts of the E A R T H.

WE have shewn how the various *Phænomena* of the Transit may be computed and estimated by PROJECTION and CALCULATION; but there remains yet the most natural and genuine Method for ascertaining the same, which is by means of the PARALLAX; and as it has not yet been considered and applied for this Purpose (*more astronomico*) we shall here explain that useful Doctrine to the Tyro in Astronomy.

For this Purpose it will be proper for him to have before him a CELESTIAL GLOBE, as he will that Way have a more clear and easy Idea of all the fundamental Principles of this Part of the Science, and which are represented in Figure 3. Plate III. Where H Z O N is the general *Meridian*; P the North Pole; Æ K the *Equinoctial*; T S L the *Ecliptic*, whole Pole is Q; C S V the Orbit of VENUS; S the Place of the SUN, very near the *ascending Node*; P S B an *Hour-Circle*; Z S N a *vertical Circle*; Q E S a *Circle of Latitude*; Q Z G another passing through the Zenith Z, and *Nonesima* Degree G; and Q I another passing through the Pole P, to the Beginning of *Cancer* at I.

This is the Position of the Globe and its Circles, for representing the *Moment of the End* of the Transit, as viewed at LONDON; and therefore the Angle $ZPS = 3^h : 17' = 49^\circ : 15'$. Also the Side $ZP = 38^\circ : 30'$, and $PS = 67^\circ : 17'$ the *Co-Declination of the Sun*, whence the Angle P S Z is found to be $45^\circ : 43'$.

Then

Then in the Triangle QSP there are known all the Sides, $PQ = 23^\circ : 30'$, $PS = 67^\circ : 17'$; SQ a Quadrant, and the Angle $PQS = 14^\circ : 24'$, the Distance of the *Sun* from *Cancer* = IS . Whence the Angle PSQ is found to be $6^\circ : 10'$; measuring the Inclination of the Circles of Latitude and Declination, or of the Ecliptic to a Parallel of the Equator, as mentioned in Page 18. of the *Dissertation*.

If from the Angle $PSZ = 45^\circ : 43'$, we take the Angle $PSQ = 6^\circ : 10'$, there will remain the Angle $ESZ = 39^\circ : 33'$, the Complement of which is the Angle $ZSG = 50^\circ : 27'$, which measures the Inclination of the vertical Circle ZS to the Ecliptic TS , at their Intersection in the Sun's Center S .

In the last Place, say, as the Sine of the Angle $ZSP = 45^\circ : 43'$ is to the Sine of the Side $ZP = 38^\circ : 30'$, so is the Sine of the Angle $ZPS = 49^\circ : 15'$ to the Sine of the Side $ZS = 41^\circ : 12'$; the Complement of which is the *Sun's* Altitude at the End of the Transit, *viz.* $SM = 48^\circ : 48'$.

Now we have elsewhere shewn (*Institution* 1899 in the *GENERAL MAGAZINE*) that the horizontal Parallax of a Planet is to its Parallax at any Altitude SM as Radius to the Sine of the apparent Distance ZS , from the Zenith Z . If then (with Dr. *Halley*) we suppose the solar Parallax to be $12'' \frac{1}{2}$, the *horizontal Parallax* of *Venus* will be $45''$ nearly (Page 6.); therefore say, as Radius to the Sine of $ZS = 41^\circ : 12'$, so is $45''$ to $30''$ nearly.

When we consider how small a Part of these Circles of *Latitude*, *Altitude*, *Ecliptic*, *Orbit* of *Venus*, &c. are contained on the *Sun's* Disk at S , there will be no Difficulty of conceiving that they may be very well represented by right Lines on the *Sun's* Disk, tho' drawn of any Size you please; therefore let the solar Disk (Figure 4.), as viewed in the Heavens at the End of the Transit, be represented by the Circle $ZLNT$, and thereon draw ZN for the Vertical, ER for the Circle of Latitude, making therewith an Angle $ESZ = 39^\circ : 33'$; and TS for the *Ecliptic*, making an Angle $ZSL = 50^\circ : 27'$, as before found; then since ER is perpendicular to TL , if we divide SR into 16 equal Parts or Minutes; or more truly 951 Seconds, and therein take $SG = 9' : 51'' = 591''$, and thro' that Point draw the right Line PQ , to contain an Angle SGP of $81^\circ : 32'$ with SR , it will truly represent the Path of *Venus* over the

the Face of the *Sun* at her next Conjunction, as seen from the Center of the Earth.

But *Venus* seen in the *Sun* from the Surface of the Earth, will appear to describe a different Path, by reason of the Parallax which will depress her to all who live *Northward*, and elevate her above the Line P Q, to all who are *Southward* of the Parallel which the *Sun* that Day describes.

From what has been said, it is also evident, that in so long a Time as the Planet takes in passing through the Chord P Q, the *Sun* must every Moment vary its Altitude, and that therefore the Parallax will be constantly altering likewise. So that supposing the Center of the Planet upon the *Sun* at C when he is in the Horizon (Figure 4.); then if C D be drawn parallel to Z N the Vertical, it will be equal to the *horizontal Parallax* of the Planet; or, in the present Case of *Venus*, we have $C D = 45''$; by the Time the Planet is advanced to the Conjunction at G, the *Sun* is risen to a certain Altitude, and the Parallax thereby diminished. Lastly, By the Time the Planet is advanced to (c) at its Egress from the *Sun*, the *Sun's* Altitude will be farther increased, and the Parallax proportionably diminished; and therefore drawing a Line through the Points C, c, it will be the *visible Path* of the Planet over the *Sun*, every-where at an unequal Distance from the true one P Q.

The two small Lines C D, and c d, being drawn parallel to the Vertical Z N, will be the Parallax, at the Beginning and End of the Transit; and those two Lines are each of them resolvable into two others; viz. C F, F D, and c f, f d; the first of which, in each Case, is perpendicular to the true Path; and the other parallel to it; and therefore at the same Time that the Parallax in Altitude C D, c d, depresses the Planet from its true Places D, d, to the visible Places C, c, it carries it backwards in its Path from D to F at the Beginning, and from d to f at the End of the Transit. And as the Motion from D to F, as also from d to f, is directly contrary to the Motion of the Planet in its Path, it *retards* the Time of its *coming on* and *going off* the Solar Disk.

What this Time is, may be found by computing the Quantity of the Lines D F, or d f. Thus for Example; let (c) be the Center of *Venus* at the Time that she touches the *Sun's* Limb internally at O; so that drawing S c O, we have c O equal to *Venus's* Semi-diameter, or $37'' \frac{1}{2}$; and S O = $914''$. Draw S M perpendicular to the

the Path PQ , bisecting it in H , then is the Angle $MSR = 8^\circ 28'$, which taken from $NSR = FSZ = 39^\circ 33'$, there will remain the Angle $NSM = 31^\circ 05'$. Now this Angle is equal to the Angle Fcd ; for fc is parallel to MS , and cd is parallel to SN .

Therefore, in the Right-angled Triangle efd , there is given the Angle $ecd = 31^\circ 05'$; and the Hypothenuse $cd = 30''$, which is the Parallax at $41^\circ 12'$, the Altitude of the *Sun* at that Time. Then say, as Radius to $30''$, so is the Sine of $31^\circ 05'$ to $15'' \frac{1}{2} = fd$. Now since *Venus* moves at the Rate of $240''$ per Hour, or $60' = 3600''$, if we say, As $240''$ of Motion is to $3600''$ of Time, so is $15'' \frac{1}{2}$ of Motion to $232'' \frac{1}{2}$ of Time, which is $3' 52'' \frac{1}{2}$, by which the Egress is retarded by the *Parallax in the Path*. After the same Manner, if the Beginning of the Transit were visible at *London*, the Quantity of FD might be computed for the *Sun's* Altitude when the Planet enters the Solar Disk, and thence the Time by which it would be retarded; or how much later it would happen at *LONDON* than if seen from the *Earth's* Center.

Besides the Effect of the Parallax upon the Motion of the Planet in its Orbit, it has another, which very much affects the Time of the Duration of the Transit, by diminishing or increasing the Chord or Path on the Disk. In the present Case, to us at *London* the Planet is depressed from the Chord PQ to the Chord VW ; on the Center S , through the Point (c) , sweep the small Arch acb , intersecting the Chord PQ in the Point (a) ; then since Kc is less than Ha by the Quantity fa , it is evident, that on this Account the *Phænomena* of the Egress will be accelerated, or they will happen sooner in the *visible Path*, as viewed from *London*, than in the *true Path*, as seen from the Center of the *Earth*, by all the Time the Planet takes to pass over the Space fa .

This Line fa is therefore to be computed; in the Right-angled Triangle GSH , we have $SG = 591''$, and the Angle $HSG = 8^\circ 28'$, whence we find $SH = 584'' \frac{1}{2}$. Again, in the small Triangle efd , we find $cf = 25'' \frac{1}{2}$; then $SH + fc = Sr = 610''$; then in the Right-angled Triangle Src , there are two Sides, Sc and Sr known, whence we find the Angle $Scr = SeH = 41^\circ 55'$, and $HSe = 48^\circ 05'$.

Then, because the Right-angled Triangles HSe , eca , and cfa , are similar, the Angles in the last are known, whence we find the Side $fa = 23''$; but $240''$ per Hour is $1''$ in $15''$ of Time; therefore

therefore $23 \times 15 = 305'' = 5' : 5''$ of Time in which *Venus* will pass over the Space $f a$, by so much therefore will the Egrefs be accelerated; but we have shewn it is retarded by the Parallax in the Path by $3' 52''$. Whence $1' 13''$ is the Time by which the Egrefs at *London* is accelerated.

In the same Manner you proceed for calculating the Times of passing over $F D$ and $A F$ in the Beginning of the Transit, where visible; and you will easily observe, if both Parts accelerate, or both retard, the Time of the Beginning or End of the Transit, or whether they are contrary to each other in that Respect. Thus for example, in the present Case, the Parallax in the Path $F D$, and that in the Perpendicular, which shortens the Path by the Space $A F$, both retard or contract the Transit at the Beginning; but at the End the Effect of one is contrary to that of the other, as we have shewn.

In the Case before us, the Beginning and End of the Transit happen in the Forenoon; but when they happen in the Afternoon, the Part of the Parallax denoted by $F D$, $f d$, accelerates the Time of the Beginning and End of the Transit, by causing the Planet to appear to come on and go off from the *Sun* sooner than it really does: The Reason of which is very obvious, with a little Reflection; especially if a Scheme be constructed like that in Figure 2. on the Western Side of the Globe, for any Place where the Transit will begin and end in the Afternoon.

It is evident from this Theory, that though the Beginning and End are both retarded, or both accelerated, when they happen before or after Noon; yet they are very unequally so: For the Beginning is retarded by both Effects of the Parallax; viz. $F D$ and $A F$; but the End is accelerated only by their Difference $d a$. And thus the Beginning is accelerated by the Sum $F D + F A$ in the Afternoon, and the End by the Difference $f a - f d$.

Therefore, in order to have the Duration of the Transit contracted as much as possible, the Observations should be made in those Places where the Beginning is before Noon, and End after; as in most Parts of the *East Indies*.

The Places should be such also as have the Beginning and End at the same Distance of Time from Noon, or at equal Altitudes of the *Sun* before and after Noon; because the Sum of the Parallaxes of two equal Altitudes being as the Sum of the Co-Sines of those Altitudes,

Altitudes, will be greater than the Sum of the Parallaxes of two unequal Altitudes of the same Number of Degrees. Thus twice the Sine of 45 Degrees from the Zenith is greater than the Sum of the Sines of 60 and 30 Degrees, as you may see in the Tables.

What we have now said relates to the Parallax FD and fd , in the Path; but then, since at a given Altitude the absolute Parallax DC ($= dc$ in equal Altitudes) is a constant Quantity, the Angle BCD must be such, that the Sum $FD + AF = AD$, shall be the greatest possible. It is evident that when *Venus's* Path PQ coincides with the Vertical ZN , we have $AF = 0$; and when it is perpendicular to the same, then $FD = 0$, and $AF = CD$; and it is easy to understand, that when a given Line DC is to be adapted in a given Angle CAD , so as to make AD a Maximum, the Angle DCA must be a *right one*; and therefore the Latitude of the Place such, that the Vertical ZN shall pass through the Center of the Planet C at the Moments of her internal Contact, at the Beginning or End of the Transit, which may easily be determined from the Principles above laid down.

For in this Case, the Angle NSM will be equal to the Angle MSO , which is $48^\circ 05'$, as we have shewn. Then if to this we add the Angle $MSR = 8^\circ 28'$, it will make the Angle $RSN = 56^\circ 33'$, which will be the Angle ESZ in Fig. 3. Then lastly, if to this you add the Angle $ESP = 60^\circ 10'$, you will have the Angle $ZSP = 62^\circ 43'$; and supposing the Transit to begin and end at $3^h 15'$ from Noon, the Angle ZPS will be $48^\circ 45'$, then in the Triangle ZSP there are two Angles, and the Side SP known, to find the Side ZP , which will be $55^\circ 10'$, and therefore $\angle Z = 34^\circ 50'$, the Latitude sought, where the Quantity $AF + FE$ (Fig. 4.) will be the greatest possible.

T H E
USE of the TRANSIT of VENUS
In INVESTIGATING the
ELEMENTS of her THEORY,
Explained and Exemplified from the
OBSERVATIONS of Mr. HORROX.

THE first and most celebrated of Observations respecting the Planet *Venus*, is that of her Passage over the *Sun's* Disk, which was observed at *Hoole*, near *Liverpool*, in *Lancashire*, by our Countryman Mr. *Horrox*, on the 24th of *November*, in the Year 1639; of which we have a particular Account in his Book afterwards published by *Hevelius* in 1673, intituled *De Venere in Sole Visa*. The *Sun* being then nearly setting in the Horizon, he prepared himself to make Observations thereon in a darkened Room, by taking his Image upon a proper Skreen, and which we have here represented by a Circle T E L R (Fig. 5. Plate III.), and being provided with proper Instruments to measure the several Parts, he observed her to be wholly within the Disk, and then to touch the *Sun's* Limb internally at A, at 15 Minutes after III. The Distance between the Center of the Planet V and that of the *Sun* S, he found to be 14' 25'', reckoning the Minutes such that the Diameter of the *Sun* contained 30', and that of the Planet 1' 10''.

The second Observation was made at 35 Minutes after III. and the Distance he then found between *Venus* and the *Sun* to be 13' 30''. And by a third Observation, at 45 Minutes after III, he found the Centers of *Venus* and the *Sun* to be just 13 Minutes apart; and the *Sun* setting at 50 Minutes after III, put an End to any further Observations.

From

From those few and imperfect Observations, *Horrox* proceeded to find the precise Time of the Conjunction of *Venus* with the *Sun*, and other Elements of Theory, in the following Manner: He supposed the *Sun's* Place to be in Π $12^{\circ} 24'$, and having calculated the Angle $A S T$ (in the Manner we have shewn from Fig. 3. 4.) or the Distance of the Point A from the Ecliptic T , which he found to be $46^{\circ} 34'$, whose Complement $A S R$, the Distance between *Venus* and the Circle of Latitude $E R$, is $43^{\circ} 26'$; and having supposed the Diameter of the *Sun* to be $31' 30''$, and that of *Venus* $1' 16''$ (reckoned of the usual Magnitude) he found the Distance between the Centers $S V$ to be $15' 7''$, and from thence the Difference of Longitude $V C$ to be $10' 24''$, and her Difference in Latitude $V H$ $10' 58''$.

As these Differences of Latitude and Longitude were only apparent, and as *Venus* was at that Time nearly in the Horizon, her Parallax by which she was depressed, was nearly equal to the Horizontal Parallax, which he made to be $52''$, supposing that of the *Sun* to be $14''$, the Difference of which is $38''$ for the Parallax of *Venus* from the *Sun*.

Thro' the Point V draw the Right Line $V O$ parallel to the vertical Circle $Z N$, and equal to $38''$, which was the Parallax of *Venus* from the *Sun*; then drawing the Line $O c$ parallel to $V c$, and $O h$ parallel to $V H$, there will be formed the Right-angled Triangle $V O I$, in which the Side $V O$ and the Angle at V are known (from what we have before delivered), and from thence he found the Side $I V$, which was the Parallax in Latitude, to be $36''$, and the Side $I O$ to be $13''$ in Longitude; so that the true Latitude of the Planet was $S c$, and the true Longitude $S h$. He likewise calculated, in the same Manner, the Difference of Latitude and Longitude for the other two Observations; so that, for the three Observations, the apparent and true Latitudes and Longitudes were as in the following Tables:

	True Long.	Ap. Long.	True Lat.	Ap. Lat.
At $3^h 15'$	$10' 37''$	$10' 24''$	$10' 22''$	$10' 58''$
3 35	9 36	9 22	10 3	10 38
3 45	9 5	8 51	9 49	10 24

Now from hence to find the Time and Place of the true Conjunction of *Venus* with the *Sun*, our Author supposed that the *Sun's* daily Motion was $1^{\circ} 1' 2''$, and that of *Venus* retrograde to be $36'$

38", whence the Sum $1^{\circ} 37' 40''$ measures the apparent Motion of *Venus* in Regard to the *Sun* in the Space of 24^h ; and therefore he used the following Analogy: As $1^{\circ} 37' 40''$ is to the Difference of Longitude between the Center of the *Sun* and *Venus* $10' 37''$, so is 24^h to $2^h 36' 30''$; which added to $3^h 15'$, the Time of the first Observation, gave the Time of the true Conjunction of *Venus* with the *Sun* to be at $5^h 51' 30''$ on the 14th of *November*, 1639.

From the First, compared with the Second and Third Observations, he found the Time of the Conjunction to be at $5^h 56' 30''$, and $5^h 59'$, and taking the Mean of all, he found it to be $5^h 55'$ for the Time of true Conjunction, and for that Time having calculated the *Sun's* Place in $\rightarrow 12^{\circ} 29' 35''$, he found, of course, the Place of *Venus* to be in the opposite Sign *Gemini* $12^{\circ} 29' 35''$.

Having calculated the apparent Latitude of *Venus* in the Space of 24^h , which he found to be $15' 40''$, he took the proportional Parts corresponding to $2^h 40'$ (the Time between the first Observation and Conjunction) and found it to be $1' 44''$, which having deducted from the true Latitude of *Venus* observed at the Time of the first Observation, there remained $8' 38''$ for the Latitude at the Conjunction; and by the other two Observations he found the same Latitude to be $8' 32''$ and $8' 24''$; and taking the Mean of all the three, he found it to be $8' 31''$.

In the next Place, to determine the Place of the Node, Mr. *Horrox*, from the given Distance of the Planet from the *Earth* and the *Sun*, and the Geocentric Latitude of *Venus* $8' 31''$, he found the Heliocentric Latitude to be $3' 7''$, and then supposing the Inclination of the Orbit of *Venus* to the *Ecliptic* to be $3^{\circ} 22'$, he found by Calculation (in the Manner we have before shewn) the Distance of the Planet from the Node at the Time of Conjunction to be $53' 10''$, which being added to the true Place of *Venus* which was before found in *Gemini* $12^{\circ} 29' 25''$, it gave the true Place of the Node in *Gemini* $13^{\circ} 22' 45''$, but very little different from the Place where *Kepler* had before found it, viz. only $1' 32'$.

Such is the Method by which Mr. *Horrox* attempted to settle the Elements of the Theory of this Planet; but having herein assumed the Diameter and Place of the *Sun*, also their Horizontal Parallaxes, different from what they are now found to be, this Theory was afterwards corrected by Mr. *Cassini*, and the true Latitude and Longitude

gitude for the Times of the Observations reduced to the Meridian of *Paris*, as in the following Table:

	True Long.	True Lat.
At 3 ^h 37' at <i>Paris</i>	10' 56'' $\frac{1}{2}$	10' 58'' $\frac{3}{4}$
3 57	9 53	10 36 $\frac{1}{2}$
4 7	9 20 $\frac{5}{8}$	10 21 $\frac{2}{5}$

By these true Differences of Latitude and Longitude he found the Time of the true Conjunction at *Paris* to be four Minutes after VII, and the true Place of *Venus* to be in *Gemini* 12° 33' 36'.

Mr. CASSINI's METHOD,

For DETERMINING the foregoing

ELEMENTS of VENUS's THEORY.

MR. *Cassini* having considered, that, in the Space of Half an Hour, the Motions of *Venus* in Latitude and Longitude were too little sensible for deducing the Elements of her Theory from thence, with sufficient Precision, he pursues another Method for that Purpose, on Supposition that the Ratio of the Distances of *Venus* from the *Sun* and the *Earth* is known; as also the Inclination of her Orbit, and the Quantity of her true Motion seen from the *Sun*.

He calculates the Place of the *Sun* in \star 12° 24' 52'', and supposes the Diameter of the *Sun* 32' 40'', and the solar Parallax 10''; then he proceeds to the Computation. The Longitude of *Venus* at the Time of the first Observation having been determined 10' 56'' $\frac{1}{2}$, and her Latitude 10' 58'' $\frac{3}{4}$; say, As the Distance of *Venus* from the *Sun* 72008, is to the Distance of *Venus* from the *Earth* 26522, so are 10' 56'' $\frac{1}{2}$ to 4' 2'', which measures the Difference of Longitude between *Venus* and the *Earth*, seen from the *Sun* at the Time of the first Observation, and which being deducted from the true Place of

of the *Earth*, which was then in Π $12^{\circ} 24' 52''$, gives the true Place of *Venus* in Π $12^{\circ} 20' 50''$, on the 4th of *December* 1639, at 33 Minutes after III. the true Time at *Paris*, New Stile.

In like manner say, as 72008 is to 26522, so are $10' 58''$ the Latitude of *Venus* seen from the *Earth* to $4' 3''$, which measure the Latitude of *Venus* seen from the *Sun* at the Time of the first Observation.

In like manner for the second Observation, viz. 35 Minutes after III. the true Time, her Place was found to be in Π $12^{\circ} 22' 5''$, and her heliocentric Latitude $3' 54'' \frac{1}{2}$. Lastly, For the Time of the third Observation, her Place in Π $12^{\circ} 22' 40''$, and her Latitude $3' 49''$.

The Latitude of *Venus* being thus found at the Times of the three Observations, we find the Place of the Node by the following Analogy. As the Tangent of the Inclination of the Orbit $3^{\circ} 23' 20''$ is to Radius, so is the Tangent of the heliocentric Latitude of the first Observation $4' 3''$, to the Sine of the Distance of *Venus* from the Node $1^{\circ} 8' 37''$.

By the same Analogy it is found, at the Time of the second Observation, to be $1^{\circ} 6' 13''$, and at the Time of the Third $1^{\circ} 4' 39''$; and adding these to the true Place of *Venus* seen from the *Sun*, we shall have the true Place of the Node, by the first Observation, in Π $13^{\circ} 29' 27''$, by the Second in $13^{\circ} 28' 18''$, and by the Third $13^{\circ} 27' 19''$; and taking a Mean of the three, we have the true Place of the Node in *Gemini* $13^{\circ} 28' 21''$ farther advanced by 6 Minutes than her Place determined by *Horrox*.

In order to find the Time and Place of the true Conjunction of *Venus* with the *Sun*, and her Latitude, we must consider that her daily Motion was then $1^{\circ} 36' 44''$, from which we must subtract the Motion of the *Sun* in 24^h , viz. $1^{\circ} 1' 0''$, and there will remain the Motion of *Venus* from the *Earth*, as viewed from the *Sun*, $35' 44''$ per Diem. Then say, as $35' 44''$ are to $4' 2''$, so are 24^h to $2^h 43'$, which being added to $3^h 37'$ (the Time of the first Observation) gives the Time of the Conjunction of *Venus* with the *Sun* $6^h 20'$, true Time at *Paris*; which reduced to the Meridian of *Liverpool*, will be $5^h 58'$, which is 3 Minutes later than the Time of Conjunction determined by *Horrox*; and calculating for the same Time, the true Place of the *Sun*, which is the same with that of *Venus*, it will be found in $\rightarrow 12^{\circ} 31' 44''$, the opposite Point

Point to which is *Gemini* $12^{\circ} 31' 44''$, which being deducted from the Place of the Node (before found) in *Gemini* $13^{\circ} 28' 45''$, we shall have the Distance of *Venus* from her Node at the Time of her Conjunction with the *Sun* $57' 1''$, by which we shall find her heliocentric Latitude to be $3' 22''$, and her Latitude seen from the *Earth* $9' 8''$, larger by only $37''$ than that which was determined by *Horrox*.

These are the general Methods by which the Theory of *Venus*'s Motion may be ascertained, and in a great measure settled, by numerous and accurate Observations made on the apparent Distances between the Centers of *Venus* and the *Sun* at the next ensuing Transit, which may be very easily measured by means of a proper HELIOMETER, so constructed that it shall give the true Position of the Ecliptic, with respect to the vertical Circle, for any given Moment of Time during the Transit, as also the apparent Latitude from the Ecliptic through the whole Duration.

We shall only add, that the Reason why *Venus* appears no oftener in the Face of the *Sun* is, because that when her Distance from the Node is about 2 Degrees, her Latitude seen from the *Sun* is about $7' 5''$, which Latitude, viewed from the *Earth*, will make 16 or 17 Minutes, and therefore exceed the Semi-diameter of the *Sun*. When she exceeds this small Distance from the Node, she will pass above or below the *Sun*'s Disk, and may be observed in those Conjunctions to have some Part of her enlightened Disk turn towards the *Earth*; more and more, in Proportion as she approaches the *Nonagesima* Degree of her Orbit; to which when she arrives, the Portion of her enlightened Surface will be the greatest of all, and equal to the Sum of her greatest and least heliocentric Latitude.

For suppose P (Figure 6.) to be the Place of the Planet in that Situation in her Orbit B D, let S be the Center of the *Sun*, and T that of the *Earth*; and through the Center of the Planet P draw the right Lines S K and T H, then will A C, a Part of the Orbit, be perpendicular to S K, and consequently determine the enlightened Hemisphere of the Planet A I G. Then draw C F parallel to T H, and it will cut off the Arch A C of the enlightened Hemisphere, for that Part which can be seen at the *Earth*; and because the Arch A I and C E are each 90 Degrees, and the Arch C I common to both; if that be deducted from the other two, it will leave the Arch A C equal to the Arch E I: but the Angle E P I is equal
to

to the Sum of the two internal Angles at T and S, or the Sum of the greatest heliocentric and geocentric Latitudes together; which therefore is equal to the Part of the enlightened Surface of the Planet A C turned towards the *Earth*.

To conclude; in case the Morning should prove cloudy, and we are thereby prevented from viewing the Rising-Sun with *Venus* on his Disk, I thought it might not be unacceptable to have an Idea thereof communicated by a Diagram (see Figure 7.); where H O represents the eastern Part of the Horizon, in which the *Sun* will rise. A D is a Part of the Ecliptic, described by the *Sun's* Center making an Angle with the Horizon of about 26 Degrees. A is the Position of the *Sun* below the Horizon, when its upper Limb just touches the Horizon in B. C the Center of the *Sun* just in the Horizon; and D its Position when the lower Limb just touches the Horizon in E, or the *Sun* is wholly risen above it. The Time in passing from A to D, or rising wholly above the Horizon, is a little more than four Minutes. The Path of *Venus* on the *Sun* is represented by (*a b*), and the Planet itself by a Black Spot, as far advanced on the *Sun's* Disk at his Rising, as is here represented.

N.B. *What further relates to the Transit of Venus is fully explained in two large Copper-Plate PRINTS, viz. one of seventeen Transits of this Planet over the Sun's Disk; the other of three Geographical Projections for the Time of the BEGINNING, MIDDLE, and END of the TRANSIT in every Part of the World; with the Uses thereof explained.*

F I N I S.

E R R A T A.

PAGE 4. for $N d \times \frac{b}{a}$ read $N : d \times \frac{b}{a}$

6. the Micrometer, *r.* a Micrometer.

9. of South, *r.* South.

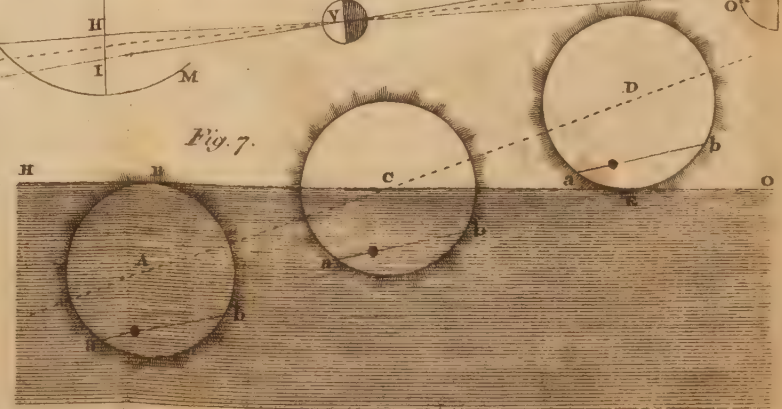
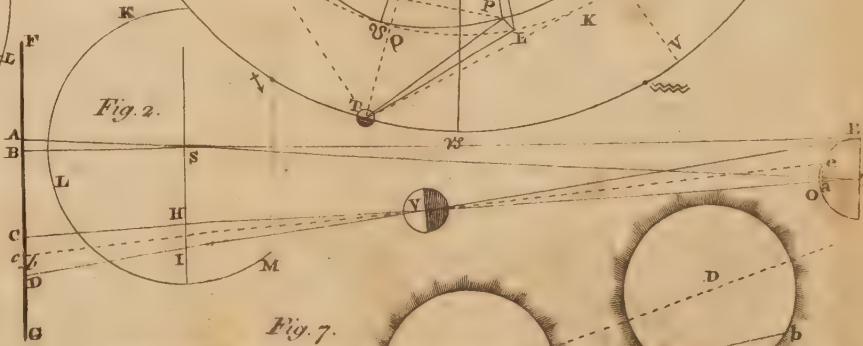
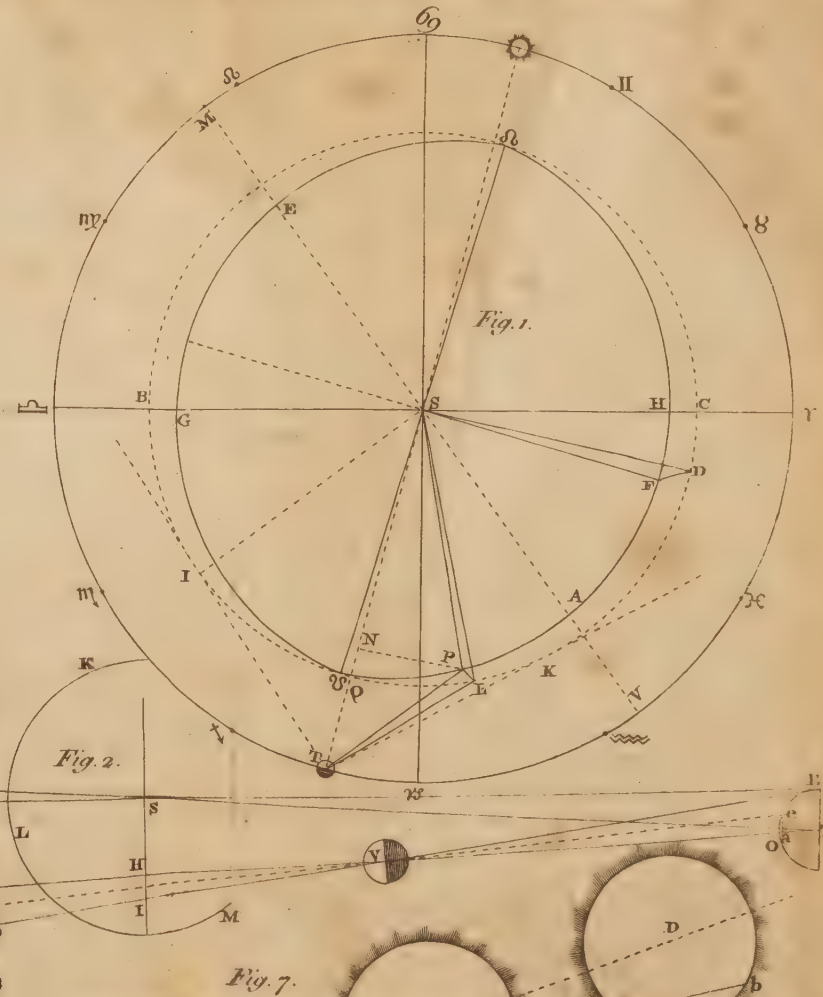
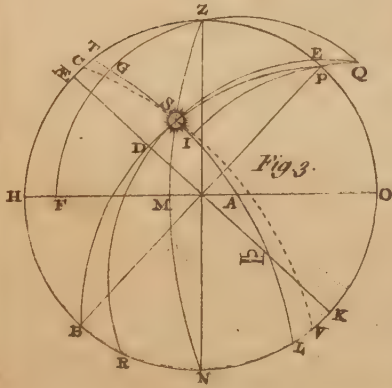
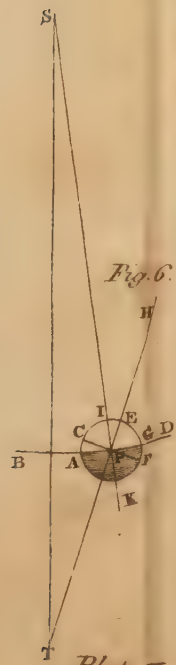
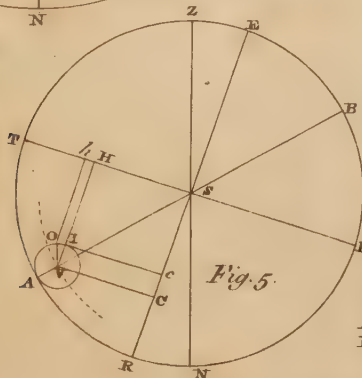
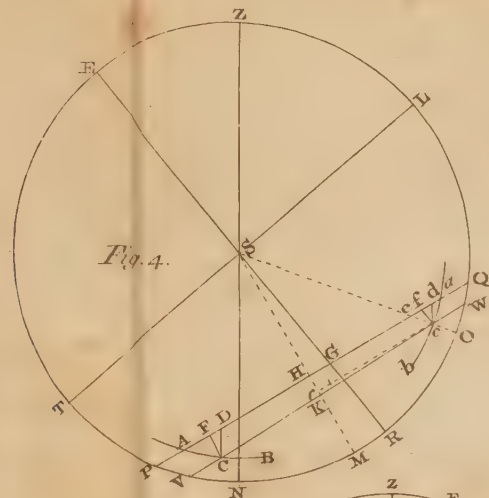
13. of the, *r.* on the.

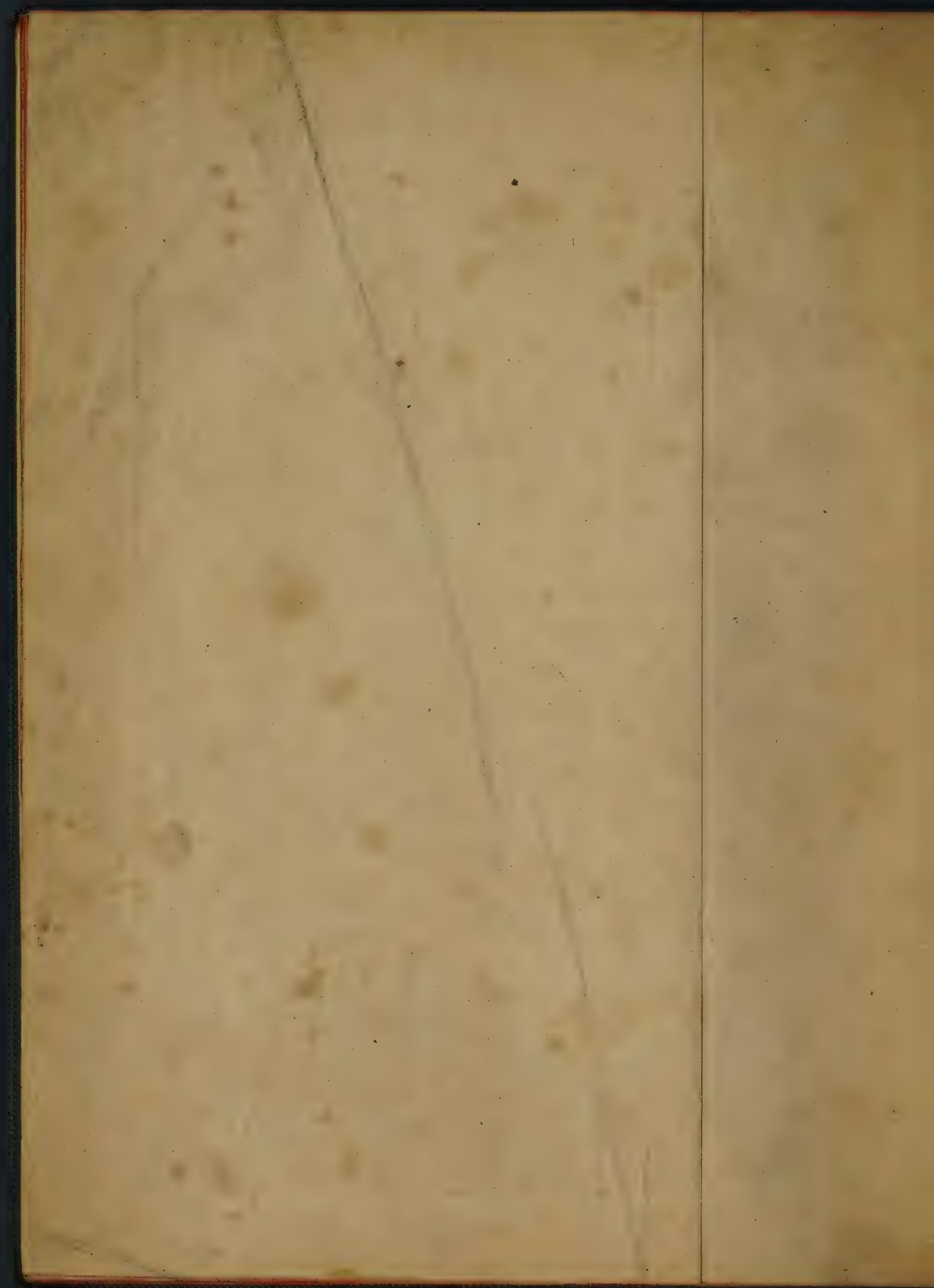
ibid. Parallax of $22^{\circ} : 30'$, *r.* Parallel of $22^{\circ} : 30'$.

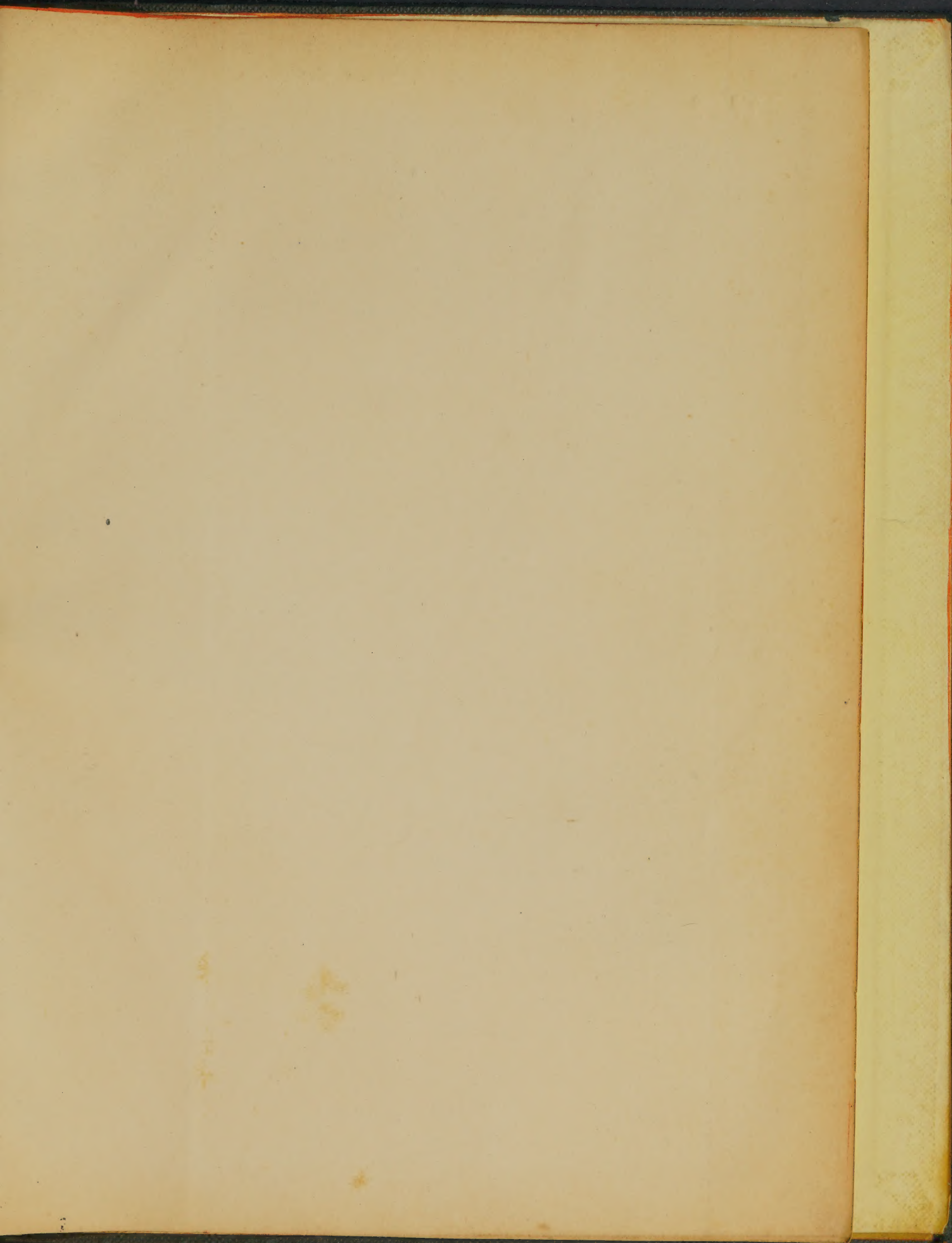
ibid. sooner at *b*, *r.* later at *b*.

17. apparent Motion of the Nodes, *r.* true Motion of the Node.

The THEORY of VENUS'S MOTION PARALLAX & other Phenomena.







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